# Hydrodynamics of arrays of OWC's devices consisting of concentric cylinders restrained in waves

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***Abstract*—This paper deals with the diffraction and the pressure-dependent-radiation problems for an array of hydro dynamically interacting OWC's devices consisting of concentric vertical cylinders that are restrained in finite depth waters and exposed to the action of regular surface waves. Such type of devices have been reported in connection with the wave energy extraction using the oscillating water column principle or in composing semi–submersible platforms for renewable electricity generation from the combined wind and wave action. Numerical and experimental results are presented concerning an array that consists of three OWC's devices made up of concentric vertical cylinders. Wave forces, air volume flow rate, inner pressure and absorbed wave power are parametrically evaluated coupled with the distance among the devices. The numerical results have been obtained using single body hydrodynamic characteristics in conjunction with the physical idea of multiple scattering to account for the interaction effects among the devices.**

***Keywords*— Oscillating water column; Wave energy; Concentric cylinders array; Air pressure oscillation; Capture width.**

## I. INTRODUCTION

The problem of the hydrodynamic interaction among neighbouring oscillating water column (OWC) devices is of a particular importance in evaluating the absorbed wave energy by a device since the hydrodynamic interaction phenomena between each member of a multi-body configuration lead in different value of the one obtained from an isolated device. Each device of the configuration scatters waves towards the others, which in turn scatter waves contributing to the excitation of the initial device and so on. In the present contribution, the total wave field around each body of the multi – body configuration is obtained by superposing the incident wave potential and various orders of successively reflected waves emanating from all the devices of the arrangement using the method of multiple scattering. The physical idea of multiple scattering was introduced by Twersky [1] in studying the acoustic scattering by an array of parallel cylinders and was applied to free-surface body - wave interaction problems by Ohkusu [2] for the case of three adjacent, floating, vertical truncated cylinders. The method is then extended by Mavrakos and Koumoutsakos [3] and Mavrakos [4] for the solution of the diffraction and radiation problems by an array of arbitrarily shaped vertical axisymmetric bodies with any geometrical arrangement and individual bodies' geometries.

Recently, some theoretical studies on arrays of OWC devices have been made to examine the amount of absorbed wave energy [5] - [9]. All these OWC devices arrays consists of a vertical cylinder partly submerged as an open - bottom chamber in which air is trapped above the inner water surface.

The present contribution deals with arrays of concentric OWC devices exposed to the action of monochromatic wave trains. The geometric configuration of the single body consists of an exterior partially immersed cylindrical structure of finite volume supplemented by an interior piston-like, free surface piercing truncated cylinder (Fig. 1). In this way, an internal free surface is formed that is enclosed between the cylinders, and pushes the dry air above through a Wells turbine system to generate power.

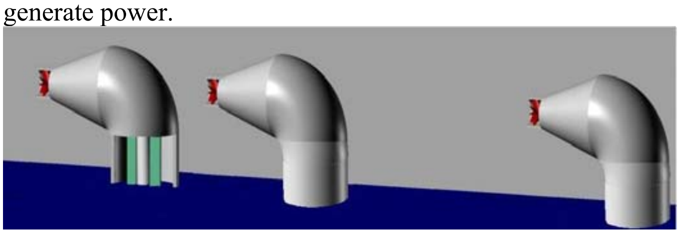

Fig. 1. Schematic representation of an array of concentric OWC devices

The fundamental hydrodynamic properties of isolated truncated hollow cylinders have been investigated some time ago ([10] - [13]) using matched axisymmetric eigenfunction expansions. Mavrakos [14] extended the formulation to the linear hydrodynamics of independently moving concentric cylinders while Mavrakos and Chatjigeorgiou [15] and Chatjigeorgiou and Mavrakos [16] investigated the corresponding second-order diffraction problem around this type of structures and treated the second-order radiation problem in the case of heaving motions of the internal cylinder. Isolated OWC devices consisting of concentric cylinders were presented by Mavrakos and Konispoliatis [17] who tested how differentiations in the device's geometry (wall thickness, draught, shape of the chamber, turbine characterises) affect the inner pressure and as a result the absorbed power by the device.

The present work aims at developing a semi–analytical method to solve the linearized diffraction and pressure radiation problems of an array that consists of $N$ number of OWC's devices and to evaluate the air volume flow rate, the inner pressure and the absorbed wave power in dependence of the distance among the devices. The numerical results for three same OWC devices placed in a row are supplemented by corresponding experimental ones that are dealing with the linearized diffraction problem (first-order exciting wave forces) of restrained multiple concentric cylinders arrangements that are open to the atmosphere (Fig. 2). They have been obtained during an experimental campaign conducted in CECHIPAR

Fig.2. Physical model of a concentric cylinder arrangement in an array of three identical bodies

research institution [18], Spain, in the framework of the European program HYDRALAB III Transnational Access Activities program that supported large European research infrastructures.

## II. FORMULATION OF THE PROBLEM

A stationary group of $N$ rigid vertical axisymmetric oscillating water column devices is considered excited by a plane periodic wave of amplitude $H/2$, frequency $\omega$ and wave number $k$ propagating in water of finite water depth $d$. The outer and inner radii of the external cylindrical body in each device $q$, $q$=1, 2,…, $N$, are $a_q, b_q$, respectively, and the distance between the bottom and sea bed is denoted by $h_{q_2}$, whereas the radius of the inner cylindrical body in each device $q$, $q$=1,2,…, $N$, is $c_q$ and the distance between its bottom and the sea bed is denoted by $h_{q_1}$ (Fig. 3). We assume small amplitude, inviscid, incompressible and irrotational flow. A global Cartesian co–ordinate system O–XYZ with origin on the sea bed and its vertical axis OZ directed positive upwards is used. Moreover, $N$ local cylindrical co-ordinate systems $\left(r_q, \theta_q, z_q\right)$, $q$ = 1, 2,…, $N$ are defined with origins on the sea bottom and their vertical axes pointing upwards and coinciding with the vertical axis of symmetry of the $q$ device (Fig. 3).

For the $p$ OWC device, $p$ = 1, 2,…, $N$, we expect the internal free surface of the device to be subjected to an oscillating pressure head $P_{in}^p$ with $P_{in}^p = \mathrm{Re}\left\{p_{in0}^p \cdot e^{-i\omega t}\right\}$, having the same frequency, $\omega$, as the incident wave.

The fluid flow around the $q$=1,2,…,$N$ device can be described by the potential function:

$$\Phi^q\left(r_q, \theta_q, z; t\right) = \mathrm{Re}\left\{\phi^q\left(r_q, \theta_q, z\right) \cdot e^{-i\omega t}\right\} \tag{1}$$

Following [19] the spatial function $\phi^q$ can be decomposed, on the basis of linear modelling, as:

$$\phi^q = \phi_0^q + \phi_7^q + \sum_{p=1}^{N} \phi_p^{qp} \tag{2}$$

Here, $\phi_0^q$ is the undisturbed incident harmonic wave velocity potential; $\phi_7^q$ is the scattered potential around the $q$ device, when it is considered fixed in waves with the duct open to the atmosphere, so that the pressure in the chamber is equal to the atmospheric one; $\phi_p^{qp}$ is the radiation potential around the $q$-th body due to time harmonic oscillating pressure head in the chamber for the $p$ device which is considered fixed in otherwise calm water.

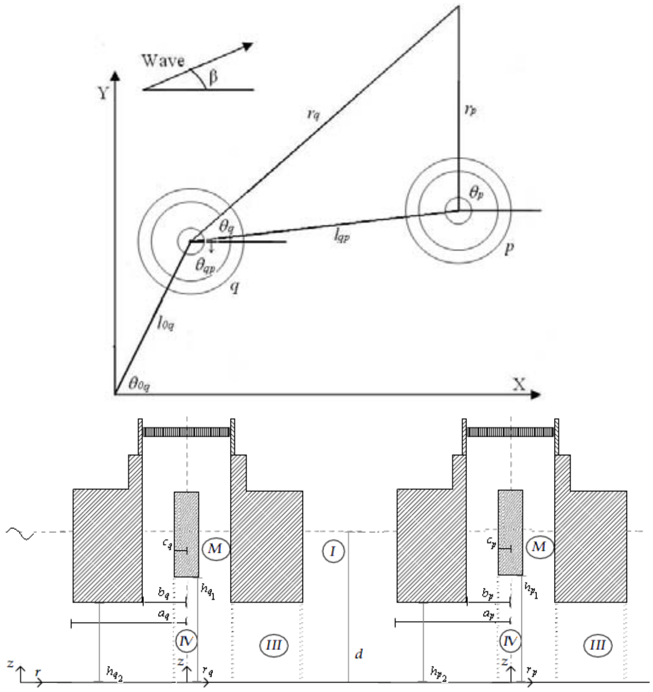


Fig. 3. Definition sketch

The potentials $\phi_j^l$ ($l \equiv q,\ qp$ ; $j$=0, 7, P; $p$=1, 2,…, $N$) are solutions of Laplace's equation in the entire fluid domain and satisfy in the entire fluid domain the following boundary conditions:

$$\omega^2 \phi_j^l - g\frac{\partial \phi_j^l}{\partial z} = \begin{cases} 0 & \text{for} \quad r_q \geq a_q; l \equiv q \text{or} qp; j = 0,7,P \\ 0 & \text{for} \quad c_q \leq r_q \leq b_q; l \equiv q; j = 0,7 \\ -\delta_{qp} i\omega p_{in0}^q / \rho & \text{for} \quad c_q \leq r_q \leq b_q; l \equiv qp; j = P \end{cases} \tag{3}$$

at the outer and inner free sea surface $\left(z = d\right)$,

$$\frac{\partial \phi_j^q}{\partial z} = 0 \quad \text{for} \quad j = 7, P \ \text{on the sea bed} \ \left(z = 0\right) \tag{4}$$

$$\frac{\partial \phi_j^q}{\partial \vec{n}} = 0 \quad \text{for} \quad j = 7, P \tag{5}$$

on the mean device's wetted surface $S_0^q$

Here $\partial(\ )/\partial\vec{n}$ denotes the derivative in the direction of the outward unit normal vector $\vec{n}$, to the mean wetted surface $S_0^q$ on the $q$–th body. Finally, a radiation condition must be imposed which states that propagating disturbances must be outgoing.

The velocity potential of the undisturbed incident wave system, $\phi_0^q$, propagating at angle β, (Fig. 3), to the positive x–axis can be expressed in the cylindrical co–ordinate frame of the $q$–th body as follows [3]:

$$\phi_0^q(r_q,\theta_q,z) = -i\omega\frac{H}{2}\sum_{m=-\infty}^{\infty} i^m \Psi_{0,m}(r_q,z)e^{im\theta_q} \tag{6}$$

Whereas

$$\frac{1}{d}\Psi_{0,m}(r_q,z) = e^{ik\ell_{oq}\cos(\theta_{oq}-\beta)}\,\frac{Z_0(z)}{dZ_0'(d)}J_m(kr_q)e^{-im\beta} \tag{7}$$

Here $J_m$ is the $m$-th order Bessel function of the first kind.

The total diffraction and the pressure radiation potential due to the oscillating pressure head in the $q$-th device, respectively, expressed in the isolated $q$–th device's cylindrical co–ordinate system $(r_q,\theta_q,z)$ can be described by:

$$\phi_D^q(r_q,\theta_q,z) = -i\omega\frac{H}{2}\sum_{m=-\infty}^{\infty} i^m \Psi_{D,m}^q(r_q,z)e^{im\theta_q} \tag{8}$$

$$\phi_P^{qq}(r_q,\theta_q,z) = \frac{p_{in0}^q}{i\omega\rho}\sum_{m=-\infty}^{\infty}\Psi_{P,m}^{qq}(r_q,z)e^{im\theta_q} \tag{9}$$

The unknown potential functions $\Psi_{D,m}^q$, $\Psi_{P,m}^{qq}$, involved in equations (8), (9) can be established through the method of matched axisymmetric eigenfunction expansions.

In doing so, the flow field around the device $q$ is subdivided in coaxial ring–shaped fluid regions, categorized by the numerals *I, III*, *M* and *IV* (Fig. 3). In each of the above regions, different series expansions of the velocity potential can be made. These are solutions of the Laplace equation in each fluid region and are selected in such a way that the corresponding conditions at the horizontal boundaries of each fluid region and, in addition, the radiation condition at infinity in the outer fluid domain are satisfied. As a result, the velocity potentials in each fluid domain fulfil a priori the kinematical boundary conditions at the horizontal walls of the bottomless cylindrical duct, the linearized condition at the free surface, the kinematical one on the sea bed, and the radiation condition at infinity.

Although the radiation potential $\varphi_P^{qq}$, around the isolated body $q$ involves only the $m$=0 term, [20], we prefer the series representations in form of equation (9) in order to obtain a similar representation with the one of the total potential, $\phi_P^{qp}$, induced around any body $q$ of the configuration due to the inner pressure in body $p$. This potential, expressed in the $q$–th body's cylindrical coordinate system $(r_q,\theta_q,z)$, includes components for all values of $m$ accounting for interference effects. Thus, it can be written as:

$$\phi_P^{qp}(r_q,\theta_q,z) = \frac{p_{in0}^p}{i\omega\rho}\sum_{m=-\infty}^{\infty}\Psi_{P,m}^{qp}(r_q,z)e^{im\theta_q} \tag{10}$$

The function $\Psi_{P,m}^{qq}$, from equation (9), has been evaluated in [17], and thus it will be no further elaborated here. Thus, the principal unknowns of the problem are the functions $\Psi_{D,m}^q$ and $\Psi_{P,m}^{qp}$.

Following the same procedure as for an array of vertical axisymmetric bodies restrained in waves [3] and as for multiple OWC devices consisted of open – bottom moonpool ducts [9], the total wave field in the outer fluid domain $I$ of body $q$, i.e. $r_q \ge a_q, 0 \le z \le d$, can be obtained by substituting equations (8)–(10) into (2) with:

$$\frac{1}{d}\Psi_{D,m}^q(r_q,z) = \sum_{j=0}^{\infty}\left[G_{Dm,j}^q\frac{I_m(a_jr_q)}{I_m(a_ja_q)} + F_{Dm,j}^q\frac{K_m(a_jr_q)}{K_m(a_ja_q)}\right]Z_j(z) \tag{11}$$

$$\Psi_{P,m}^{qp}(r_q,z) = \delta_{qp}\Psi_{P,m}^p(r_p,z) + \sum_{j=0}^{\infty}\left[G_{Pm,j}^{qp}\frac{I_m(a_jr_q)}{I_m(a_ja_q)} + F_{Pm,j}^{qp}\frac{K_m(a_jr_q)}{K_m(a_ja_q)}\right]Z_j(z) \tag{12}$$

Whereas:

$$\Psi_{P,m}^p(r_p,z) = \sum_{j=0}^{\infty}F_{Pm,j}^p\frac{K_m(a_jr_p)}{K_m(a_ja_p)}Z_j(z) \tag{13}$$

and

$$G_{\ell m,j}^v = \sum_{s=1}^{\infty}{}^sG_{\ell m,j}^v\,,\ F_{\ell m,j}^v = \sum_{s=1}^{\infty}{}^sF_{\ell m,j}^v, \ell = D,P \quad v = q,qp \tag{14}$$

Here ${}^sF_{Dm,j}^q$ is the $s$–th order scattering coefficients obtained through the solution of the respective order of diffraction problem –described by the velocity potential ${}^s\phi_D^q$ – in the co–ordinate frame of body $q$; ${}^sF_{Pm,j}^{qp}$ is the $s$–th order scattered wave coefficients from the remaining ($q \neq$ p) open – duct devices obtained through the solution of pressure radiation problem – ${}^s\phi_P^{qp}$ – in the co–ordinate frame of body $q$; and

$${}^sG_{Dmj}^q = \sum_{p=1}^{N}(1-\delta_{pq})\sum_{v=-\infty}^{\infty}i^{m+v}\frac{K_{v-m}(a_j\ell_{pq})I_m(a_ja_q)}{K_v(a_ja_q)}{}_{s-1}F_{Dv,j}^p e^{i(v-m)\theta_{pq}}, s \ge 2 \tag{15}$$

represents the contribution of the $s$-th order incident wave potential to the total diffracted wave field around the body $q$ (see eq. 8), originated from the (s-1) orders of scattered wave fields from the rest bodies ($p$ = 1, 2, …, N; $p \neq$ q) of the array. Especially, for the first order of interaction, $s$=1, the undisturbed incident wave potential, ${}^1\phi_0^q$, is given by equation (7). Moreover, for the pressure radiation problem, it holds:

$${}^sG_{Pmj}^{qp} = \sum_{\ell=1}^{N}(1-\delta_{\ell q})\sum_{v=-\infty}^{\infty}i^{m+v}\frac{K_{v-m}(a_jl_{\ell q})I_m(a_ja_q)}{K_v(a_ja_q)}{}_{s-1}F_{Pv,j}^{\ell p}e^{i(v-m)\theta_{\ell q}} \tag{16}$$

Here $I_m$ $K_m$ are the $m$–th order modified Bessel function of first and second kind, respectively, and $Z_j(z)$ are orthonormal functions in $[0,d]$ defined as follows:

$$Z_0(z) = N_0^{-1/2}\cosh(kz), \quad j = 0 \tag{17}$$

$Z_0'(d)$ being its derivative at $z=d$,

$$Z_j(z) = N_j^{-1/2}\cos\left(a_j z\right), \quad j \geq 1 \tag{18}$$

With

$$N_0 = \frac{1}{2}\left[1 + \frac{\sinh(2kd)}{2kd}\right], \; N_j = \frac{1}{2}\left[1 + \frac{\sin(2a_j d)}{2a_j d}\right] \tag{19}$$

Here $k$ is related to $\omega$ through the dispersion equation:

$$\omega^2 = kg\tanh(kd) \tag{20}$$

Whereas $a_j$ are the real solutions of the equation:

$$\frac{\omega^2}{g} + a_j\tanh(a_j d) = 0 \tag{21}$$

The first term in equation (12) represents the isolated device wave field around the $p$ body due to its own internal pressure variation, the second term denotes the incident wave fields on body $q$ emanating from the scattered fields of the remaining devices considered open and the last term is the scattered wave filed around the $q$ –th device.

The corresponding expressions for the total velocity potential in the *III*, i.e. $b_q \leq r_q \leq a_q, 0 \leq z \leq h_{q_2}$ , *M*, i.e. $c_q \leq r_q \leq b_q, 0 \leq z \leq d$ , and *IV*, i.e. $0 \leq r_q \leq c_q, 0 \leq z \leq h_{q_1}$ , fluid regions can be calculated with:

$$\mu \cdot \Psi_{\ell,m}^{III,v} = \sum_{n=0}^{\infty} \varepsilon_n \left[\left(F_{\ell m,n}^{III,v} + \delta_{qp}F_{P0,n}^{III,p}\right)R_{mn}^{III} + \left(F_{\ell m,n}^{*III,v} + \delta_{qp}F_{P0,n}^{*III,p}\right)R_{mn}^{III*}\right] \cos\left(\frac{n\pi z}{h_{q_2}}\right) \tag{22}$$

$$\mu \cdot \Psi_{\ell,m}^{M,v} = \delta_{qp}g_{P,m}^{p}(r_p,z) + \sum_{n=0}^{\infty}\left[\left(F_{\ell m,n}^{M,v} + \delta_{qp}F_{P0,n}^{M,p}\right)R_{mn}^{M} + \left(F_{\ell m,n}^{*M,v} + \delta_{qp}F_{P0,n}^{*M,p}\right)R_{mn}^{M*}\right]Z_n(z) \tag{23}$$

$$\mu \cdot \Psi_{\ell,m}^{IV,v} = \sum_{n=0}^{\infty} \varepsilon_n \left(F_{\ell m,n}^{IV,v} + \delta_{qp}F_{P0,n}^{IV,p}\right)R_{mn}^{IV}\cos\left(\frac{n\pi z}{h_{q_1}}\right) \tag{24}$$

whereas $\ell = D, P$ ; $v = q, qp$ ; $\mu = 1/d$ for diffraction problem and $\mu = 1$ for pressure radiation problem, and

$$F_{\ell m,n}^{III,v} = \sum_{s=1}^{\infty} {}^{s}F_{\ell m,n}^{III,v}, \; F_{\ell m,n}^{*III,v} = \sum_{s=1}^{\infty} {}^{s}F_{\ell m,n}^{*III,v},$$

$$F_{\ell m,n}^{M,v} = \sum_{s=1}^{\infty} {}^{s}F_{\ell m,n}^{M,v}, \; F_{\ell m,n}^{*M,v} = \sum_{s=1}^{\infty} {}^{s}F_{\ell m,n}^{*M,v} \quad \text{and}$$

$$F_{\ell m,n}^{IV,v} = \sum_{s=1}^{\infty} {}^{s}F_{\ell m,n}^{IV,v} \tag{25}$$

$$R_{mn}^{III}(r_q) = \frac{K_m\left(\frac{n\pi b_q}{h_{q_2}}\right)I_m\left(\frac{n\pi r_q}{h_{q_2}}\right) - I_m\left(\frac{n\pi b_q}{h_{q_2}}\right)K_m\left(\frac{n\pi r_q}{h_{q_2}}\right)}{I_m\left(\frac{n\pi a_q}{h_{q_2}}\right)K_m\left(\frac{n\pi b_q}{h_{q_2}}\right) - I_m\left(\frac{n\pi b_q}{h_{q_2}}\right)K_m\left(\frac{n\pi a_q}{h_{q_2}}\right)}$$

$$R_{mn}^{III*}(r_q) = \frac{I_m\left(\frac{n\pi a_q}{h_{q_2}}\right)K_m\left(\frac{n\pi r_q}{h_{q_2}}\right) - K_m\left(\frac{n\pi a_q}{h_{q_2}}\right)I_m\left(\frac{n\pi r_q}{h_{q_2}}\right)}{I_m\left(\frac{n\pi a_q}{h_{q_2}}\right)K_m\left(\frac{n\pi b_q}{h_{q_2}}\right) - I_m\left(\frac{n\pi b_q}{h_{q_2}}\right)K_m\left(\frac{n\pi a_q}{h_{q_2}}\right)}$$

$$R_{mn}^{M}(r) = \frac{I_m(a_n c_q)K_m(a_n r) - K_m(a_n c_q)I_m(a_n r)}{I_m(a_n b_q)K_m(a_n c_q) - I_m(a_n c_q)K_m(a_n b_q)}$$

$$R_{mn}^{M*}(r) = \frac{I_m(a_n b_q)K_m(a_n r) - K_m(a_n b_q)I_m(a_n r)}{I_m(a_n b_q)K_m(a_n c_q) - I_m(a_n c_q)K_m(a_n b_q)}$$

$$R_{mn}^{IV}(r_q) = I_m\left(\frac{n\pi r_q}{h_{q_1}}\right) \Big/ I_m\left(\frac{n\pi c_q}{h_{q_1}}\right) \tag{26}$$

$$g_{P,m}^{p}(r_p,z) = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases} \tag{27}$$

and $\varepsilon_n$ is the Neumann's symbol: $\varepsilon_0 = 1$ , $\varepsilon_n = 2$ , $n \geq 1$ .

## III. VOLUME FLOW

During the water oscillation inside the chamber of $q$ device $q$=1,2,…,$N$, the dry air above the free surface is being pushed through a Wells turbine. The time dependent volume flow produced by the oscillating internal water surface is given by:

$$Q^q\left(r_q,\theta_q,z;t\right) = \mathrm{Re}\left[q^q\left(r_q,\theta_q,z\right)\cdot e^{-i\omega t}\right] \tag{28}$$

where:

$$q^q = \iint_{S_i^q} u_z dS_i = \iint_{S_i^q} u(r_q,\theta_q,z=d)r_q dr_q d\theta_q = \iint_{S_i^q} \frac{\partial \phi^q}{\partial z} r_q dr_q d\theta_q \tag{29}$$

Here $u_z$ denotes the vertical velocity of the water surface, and $S_i^q$ the inner water surface of the $q$ device. It proves convenient to decompose the total volume flow, $q^q$ , of the $q$ device, into two terms associated with the diffraction, $q_D^q$ , and the pressure–dependent radiation problem, $q_P^q$ , as follows:

$$q^q = q_D^q + p_{in0}^q q_P^q = q_D^q - p_{in0}^q\left(B^p - iC^P\right) \tag{30}$$

whereas $B^q$ and $C^q$ are the corresponding radiation conductance and susceptance, respectively. Assuming uniform pressure distribution inside the chamber, it can be shown that, even though all $m$-modes terms affect the values of the diffraction and radiation potentials, by substituting those potentials in equation (29) only the $m = 0$ modes contribute to $q_P^q$ and $q_D^q$ , as in an isolated device [21].

The total volume flow in each body $q$ of the configuration is proportional to the pressure of each of the remaining $n$, $n = 1,2,...,N, n \neq q$ , bodies of the array, thus:

$$q^q = q_D^q + p_{in0}^q \sum_{n=1}^{N} q_P^n \frac{p_{in0}^n}{p_{in0}^q} \tag{31}$$

## IV. AIR PRESSURE CALCULATION

Assuming that the Wells turbine is placed in each duct of the $q$ devices, between the chamber and the outer atmosphere, and it is represented by a pneumatic admittance $\Lambda^q$ , then the volume flow in $q$ device is equal to:

$$Q^q(t) = \Lambda^q \cdot P_{in}^q(t) \tag{32}$$

The mass flow rate of air though the turbine can be written as [22]:

$$\frac{\dot{m}^q}{\rho_a} = Q^q(t) - \frac{V_0^q}{\gamma \cdot P_a}\frac{dP_{in}^q(t)}{dt} \tag{33}$$

Here, $V_0^q$ is the air volume in undisturbed conditions in device $q$, $q$=1,2,…,$N$, $\gamma = 1.4$ is the adiabatic constant and $P_a$ is the atmospheric pressure.

According to [23] and [24], for the Wells turbine can be obtained:

$$q^q = \left[ \frac{KD}{\rho_a N} + (-i\omega)\frac{V_0^q}{\gamma P_a} \right] p_{in0}^q \quad (34)$$

Whereas $K$ is constant for a given turbine geometry (independent of turbine size or rotational speed), $D$ is turbine rotor diameter, $N$ is the rotational speed (radians per unit time) and $\rho_a$ is the atmospheric density.

The imaginary part of $\Lambda^q$ may be of some importance in a full-scale OWC device, but it is usually negligible in down-scaled laboratory model experiments [25].

## V. WAVE FORCES

The various forces acting on the $q$ oscillating water column device can be calculated from the pressure distribution given by the linearised Bernoulli's equation:

$$P(r_q, \theta_q, z; t) = -\rho \frac{\partial \Phi^q}{\partial t} = i\omega\rho\phi^q \cdot e^{-i\omega t} \quad (35)$$

Whereas $\phi^q$ is the $q$ devices' velocity potential in each fluid domain *I*, *III*, *M* and *IV*.

The hydrodynamic forces and moments acting on the body $q$, $q$=1,2,…,$N$ can be calculated by integration the pressure over the mean wetted surface $S_0^q$ using the relation:

$$F_i^q = f_i^q \cdot e^{-i\omega t} = -\int\int_{S_0^q} i\omega\rho\phi_D^q \cdot e^{-i\omega t} \cdot n_i^q dS \quad (36)$$

In addition, the hydrodynamic reaction forces and moments $F_i^{qp}$ acting on the device $q$ in the $i$–th direction due to pressure in the $p$ device with inner pressure $p_{in0}^p$ and frequency $\omega$, can be obtained:

$$F_i^{qp} = f_i^{qp} \cdot e^{-i\omega t} = -\int\int_{S_0^q} i\omega\rho\phi_p^{qp} \cdot e^{-i\omega t} \cdot n_i^q dS \quad (37)$$

Whereas $n_i^q$ are the generalized normal components defined by:

$$n^q = \left(n_1^q, n_2^q, n_3^q\right),\ r^q \times n^q = \left(n_4^q, n_5^q, n_6^q\right) \quad (38)$$

and $r^q$ is the position vector of a point on $S_0^q$.

Furthermore, the complex force $f_i^{qp}$ may be written in the form:

$$f_i^{qp} = \omega^2 \left( e_i^{qp} + \frac{i}{\omega} d_i^{qp} \right) \cdot p_{in0}^p \quad (39)$$

Whereas $e_i^{qp}, d_i^{qp}$ are the added mass and dumping coefficients, respectively.

## VI. ABSORBED POWER

The averaged value of the power absorbed from the waves over one wave period from the device $q$ is obtained from [25]:

$$W^q = \frac{1}{2}\mathrm{Re}\left[ \overline{q_D^q} p_{in0}^q \right] - \frac{1}{2}\overline{p_{in0}^q} B^q p_{in0}^q = \frac{1}{4}\left( \overline{q_D^q} p_{in0}^q + q_D^q \overline{p_{in0}^q} \right) - \frac{1}{2}\overline{p_{in0}^q} B^q p_{in0}^q \quad (40)$$

The capture width $\ell^q$ of the $q$ device is the ratio of the power absorbed by the device to the available power per unit crest length of the incident wave [26], i.e.

$$\ell^q = 2 \cdot W^q \big/ (\rho g (H/2)^2 C_g) \quad (41)$$

$C_g$ being the group velocity of the incident wave.

The absorbed power (equation (40)) takes a maximum value of [27]:

$$W_{\max}^q = \overline{q_D^q} \left(B^q\right)^{-1} q_D^q \big/ 8 \quad (42)$$

corresponding to an optimum inner pressure head:

$$p_{in0_{optim}}^q = \left(B^q\right)^{-1} q_D^q \big/ 2 \quad (43)$$

The resulting maximum capture width is then given by equation (41).

## VII. NUMERICAL RESULTS

The calculation of the coefficients $F_{jm,i}^I, F_{jm,q}^{III}$, $F_{jm,q}^{*III}, F_{jm,i}^M, F_{jm,i}^{*M}, F_{jm,q}^{IV}$, $j$=$D$,$P$ is the most significant part of the numerical procedure, because they affect the accuracy of solution. For the present calculations, in the *I-th* and the *M-th* ring element $i = 30$ terms were used, while for the *III-th* and *IV-th* ring element $q = 40$. In addition, the presented results were obtained for five order interactions.

Firstly, we examine an array of three same OWC devices placed in a row, for $a_q = 6b_q/5$, $c_q = 4b_q/5$ and $d = 10b_q$ (for definitions see figure 3). The draughts of the outer and inner cylindrical body are $b_q$ and $4b_q/5$, respectively, and the distance among the devices (Fig. 4) is $\ell = 4b_q$. The incident wave is propagating to the positive x–axis at θ=0° angle, the origin of the Cartesian co–ordinate system is on the sea bed and its vertical axis directed positive upward coinciding with the vertical axis of the second device local coordinate system. In figure 5 the horizontal (acting on x axis) and vertical exciting forces on the outer and inner cylindrical body of each OWC device of the configuration is depicted. The numerical results were tested against the experimental ones obtained in CECHIPAR research institute experimental campaign.

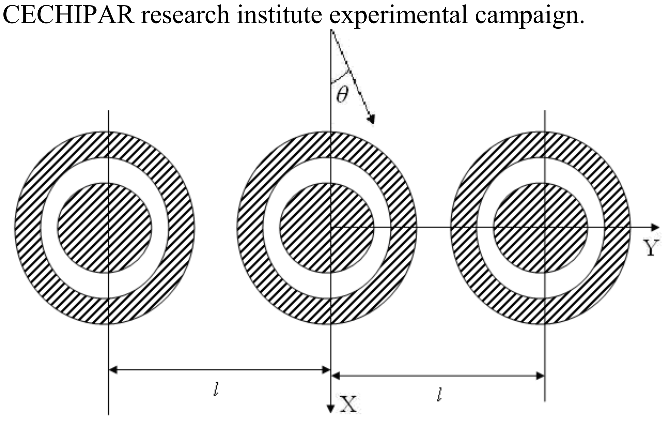


Fig. 4. Array of three identical OWC devices placed in a row

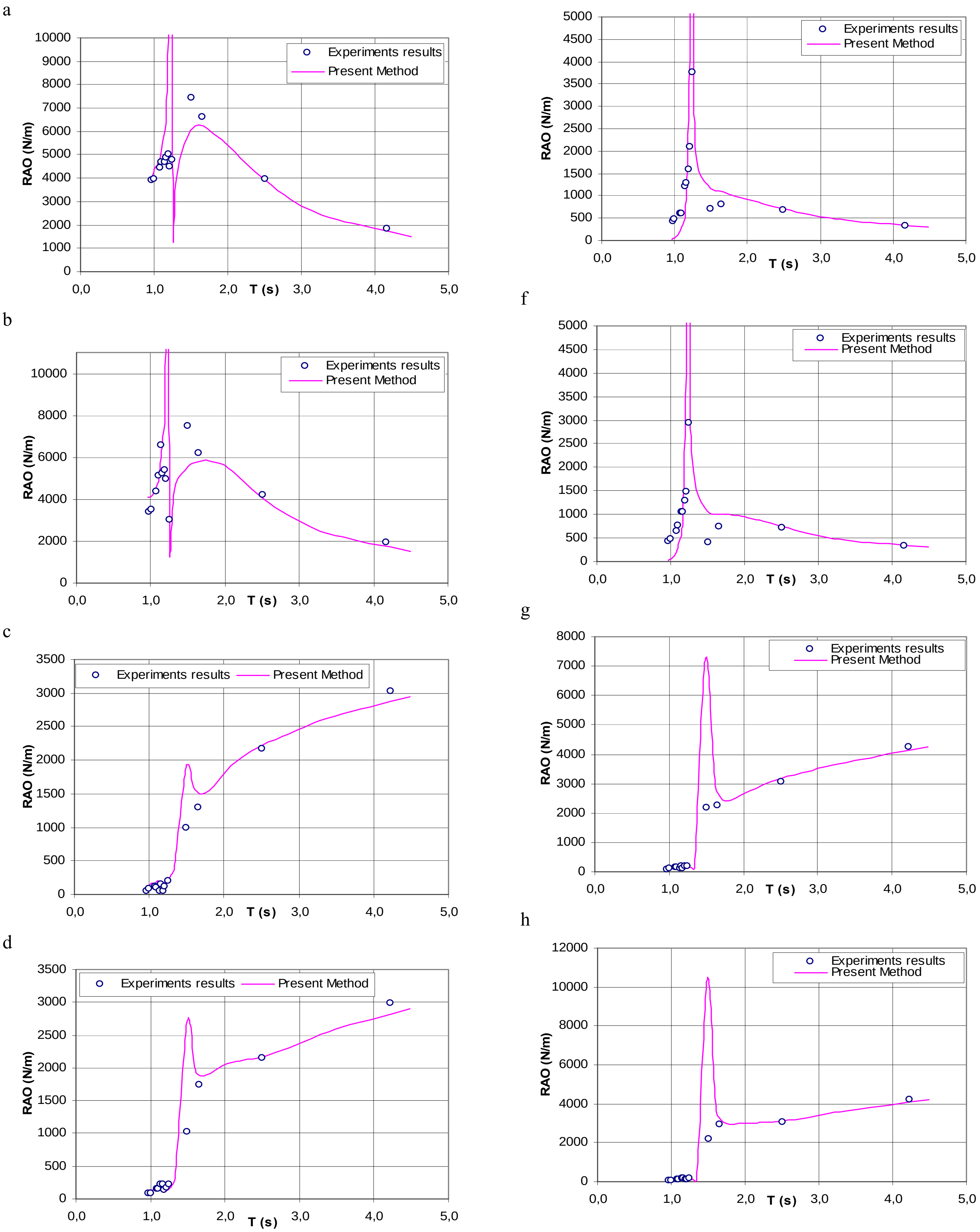


Fig. 5. Comparison of the horizontal and vertical exciting wave forces on outer and inner cylindrical body of each OWC device of the configuration. (a)-(b) correspond to the horizontal exciting wave force on outer cylindrical body of side and middle OWC devices respectively; (e)-(f) correspond to the horizontal exciting wave force on the inner cylindrical body of side and middle OWCs respectively; (c)-(d) correspond to the vertical exciting wave force on outer cylindrical body of side and middle OWCs respectively, and (g)-(h) correspond to the vertical one on inner cylindrical body of side and middle OWCs respectively.

Next, we examine how the distance among the devices in an array configuration affects the maximum capture width. In figure 6 the value of $v = \sum_{q=1}^{N} \ell_{\max}^{q} \Big/ N \dfrac{\lambda}{2\pi}$, $N$=3, is being plotted for the same as the above configuration, i.e. $a_q = 6b_q/5$, $c_q = 4b_q/5$ and draughts of the outer and inner cylindrical body $b_q$ and $4b_q/5$, respectively. The water depth is $d = 20b_q$, while the distances among the devices are $\ell = 4b_q, 8b_q, 32b_q$. It is reminded that $\lambda/2\pi$ is the maximum capture width by an isolated heaving device [25].

Next, we studied how each device's draught affects the maximum capture width. In figure 7 the draught of each device varies while the distance among the devices is kept the same, i.e. $(d - h_{q_2}) = b_q, 2b_q, 3b_q, 5b_q$ and $(d - h_{q_1}) = 4b_q/5$, $9b_q/5, 14b_q/5, 24b_q/5$ (for definitions see figure 3). Here the value of $b_q \cdot v/(d - h_{q_2})$ is plotted for the same configuration as the above of OWC devices. The rest geometric characteristics for the single body are kept the same i.e. $a_q = 6b_q/5$, $c_q = 4b_q/5$, $d = 20b_q$, $\ell = 4b_q$.

Finally, we examined how the inner cylindrical body's radius affects the maximum capture width. In figure 8 the value of $(b_q - c_q) \cdot v/b_q$ is plotted for various inner cylindrical body's radiuses in order to compare its values with those obtained from an array of OWC devices without central cylindrical body, i.e. moonpool OWC devices. The array of moonpool OWC devices were placed in a row with same wave propagation as the above. The geometric characteristics for the single moonpool body are $a_q = 6b_q/5$, $d = 20b_q$, $h_{q_2} = 19b_q$ and for the single concentric one are: $a_q = 6b_q/5$, $d = 20b_q$, $h_{q_2} = 19b_q$, $h_{q_1} = 96b_q/5$ while $c_q = 4b_q/5$, $3b_q/5, 2b_q/5, b_q/5$.

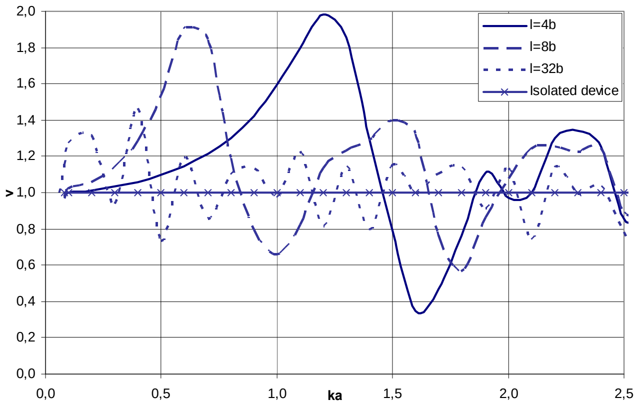


Fig. 6. Value of $v = \sum_{q=1}^{3} \ell_{\max}^{q} \Big/ \dfrac{3\lambda}{2\pi}$ versus $ka_q$ for three OWC devices placed in a row for various distances among them.

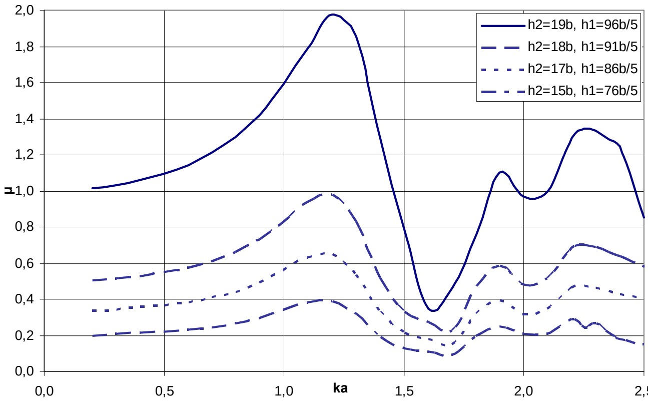


Fig. 7. Value of $\mu = \sum_{q=1}^{3} b_q \ell_{\max}^{q} \Big/ (d - h_{q_2}) \dfrac{3\lambda}{2\pi}$ versus $ka_q$ for three OWC devices placed in a row for various draughts of each devise.

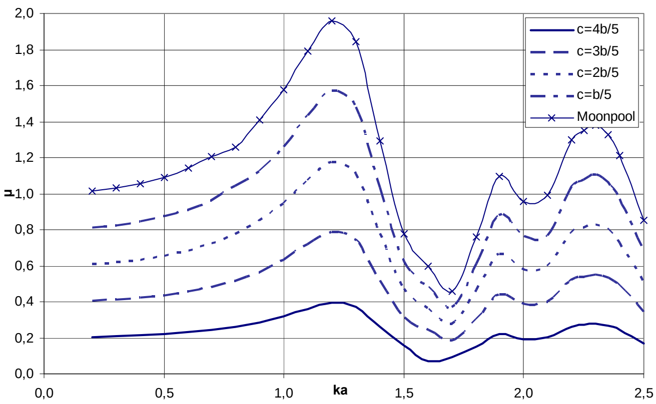


Fig. 8. Value of $\mu = \sum_{q=1}^{3} (b_q - c_q) \ell_{\max}^{q} \Big/ b_q \dfrac{3\lambda}{2\pi}$ versus $ka_q$ for three OWC devices placed in a row for various radiuses of the inner cylindrical body.

## VIII. CONCLUSIONS

A semi-analytical method has been developed to solve the pressure radiation problem around an array of multiple interacting oscillating water column devices consisting of vertical concentric cylinders. The exciting wave forces acting on each body of a three device arrangement, placed in a row, were tested against measurments of an experimental campaign conducted in CECHIPAR research institute with good agreement. In addition, the effect of the distances among the devices of the array on the ratio, v, of the total average capture width per unit crest length of the incident wave was plotted. The value of v tends to the value of the isolated device – unity for all incident wavelengths – while the distance among the devices increases. Moreover, the effect of each devices draught on the maximum capture width was tested. From the depicted results the total average of the capture width is decreasing while the draught of each device increases. Finally, it was examined how inner cylindrical body's radius affects the maximum capture width. As the radius of the inner cylindrical body increases, by keeping the rest geometric characteristics of the device constant, the maximum capture width is decreasing, since the inner water surface is also decreasing.


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