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HYDRODYNAMICS AND POWER ABSORPTION CHARACTERISTICS OF FREE FLOATING AND MOORED ARRAYS OF OWC'S DEVICES

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ABSTRACT

This paper deals with the diffraction, the pressure– and motion–dependent–radiation problems for an array of hydro–dynamically interacting OWC's devices consisting of concentric vertical cylinders that are floating in finite depth waters and exposed to the action of regular surface waves. The devices are either floating independently or as a unit assuming a platform. Such type of devices have been reported in connection with the wave energy extraction using the oscillating water column principle or in composing semi – submersible platforms for renewable electricity generation from the combined wind and wave action, see Fig.1, (Aubault et al., 2011; Mavrakos et al., 2011). The wave action causes the captured water column to oscillate in the chamber, compressing and decompressing the air above the inner water surface. As a result, there is an air flow moving forwards and backwards through a turbine coupled to an electric generator.

INTRODUCTION

Recent research efforts (Nader et al., 2012; Konispoliatis & Mavrakos, 2013a, b) have been dealt with the evaluation of the hydrodynamic and power absorption characteristics of arrays of interacting OWC's devices consisting of circular or concentric cylindrical champers. The devices were assumed restrained in regular waves. The diffraction– and the pressure induced radiation– problems have been analytically treated and relevant results presented. In the present contribution, the methodology is extended to include the motion–dependent radiation problem and to solve the pressure–motion equations of multiple floating OWC's arrangements.

The problem of the hydrodynamic interaction among neighbouring floating devices is of a particular importance since the hydrodynamic characteristics of each member of a multi–body configuration may differ from the ones obtained from an isolated device due to interaction phenomena with neighbouring bodies. Each device of the array scatters waves which excite the remaining bodies which in turn respond to this excitation and scatter waves contributing to the excitation of the initial body and so on. Therefore it is clearly important to be able to estimate the interactions between the devices of the array in order to create a background theory in developing arrays of OWC devices. In that context, the physical idea of multiple scattering is being exploited in the fashion that was initially presented in Mavrakos & Koumoutsakos (1987) and Mavrakos (1991) work.

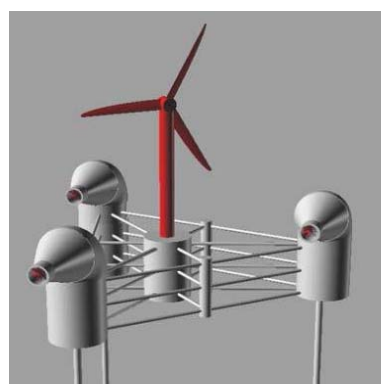

Figure 1: Wind & wave action submersible platform, consisting of three OWC devices

In the present contribution, we consider a system of N OWC's devices which can oscillate about their mean equilibrium position moving either independently or as a unit. The geometric configuration of each device consists of an exterior partially immersed toroidal oscillating chamber of finite volume supplemented by a concentric interior piston–like truncated cylinder, see Figs. 2, 3. In each device, the oscillating inner air pressure is a function of the motions of each member of the array. In the case of a rigid multi–OWC's device configuration, the individual chambers' pressures depend from the motion components of the entire structure. Therefore the absorbed power of each of the device of the multi–component structure are presented as a function of the six modes of motion of the structure and compared with the ones obtained from the individual freely floating members.

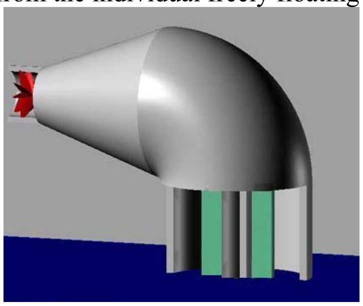

Figure 2: Floating OWC device consisting of a concentric vertical cylinder

Moreover, the influence of the neighbouring bodies on the total wave field around each member of the OWC's array along with their effect on the array's power efficiency is examined. In exploiting the multiple scattering approach to evaluate the hydrodynamic interaction effects among the members of the multi – body configuration, the single isolated body hydrodynamic characteristics are required. Here the method of matched axisymmetric eigenfunction expansion as it was implemented either for truncated vertical cylinders or for arbitrarily shaped vertically axisymmetric compact bodies (Garret, 1971; Yeung, 1981; Mei, 1983; Kokkinowrachos et al., 1986) has been used. According to this method, the flow field around the bodies is subdivided in coaxial ring–shaped fluid regions, categorized by the numerals *I, III*, *M* and *IV* (Fig. 3) in each of which appropriate series representations of velocity potential can be established.

Finally, the effect of moorings on the developed pressure head inside each device of the array is investigated thoroughly together with the motion dynamics of the floating arrangement.

DESCRIPTION OF THE HYDRODYNAMIC PROBLEM

We consider a group of N floating vertical axisymmetric oscillating water column devices that is excited by a plane periodic wave of amplitude $H/2$, frequency ω and wave number k propagating in water of finite water depth d. The outer and inner radii of each device's chamber q, q=1, 2,…, N, are denoted by a_q, b_q, respectively, whereas the distance between the bottom of the q device and the sea bed is denoted by h_q. The radius of the interior concentric cylindrical body in each device q, is denoted by $b_{1,q}$ and the distance between its bottom and the sea bed is $h_{1,q}$ (Fig. 3). It is assumed small amplitude, inviscid, incompressible and irrotational flow, so that linear potential theory can be employed. A global Cartiesian co–ordinate system O–XYZ with origin on the sea bed and its vertical axis OZ directed positive upwards is used. Moreover, N local cylindrical co-ordinate systems $\left(r_q, \theta_q, z_q\right)$, q = 1, 2,…, N are defined with origins on the sea bottom and their vertical axes pointing upwards and coinciding with the vertical axis of symmetry of the q device.

The fluid flow around the q=1,2,…,N device can be described by the potential function:

$$\Phi^q\left(r_q, \theta_q, z; t\right) = \mathrm{Re}\left\{\phi^q\left(r_q, \theta_q, z\right) \cdot e^{-i\omega t}\right\} \tag{1}$$

Following Falnes (2002) the spatial function ϕ^q can be decomposed, on the basis of linear modelling, as:

$$\phi^q = \phi_0^q + \phi_7^q + \sum_{p=1}^{N} \sum_{j=1}^{6} \dot{x}_{j0}^p \cdot \phi_j^{qp} + \sum_{p=1}^{N} p_{in0}^p \cdot \phi_P^{qp} \tag{2}$$

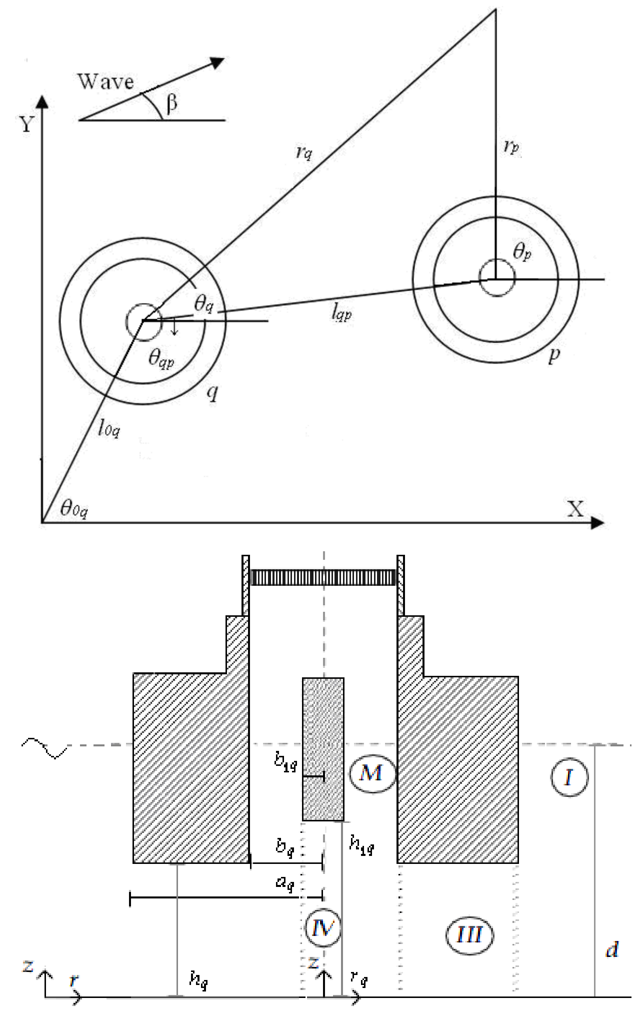

Figure 3: Definition sketch of the q OWC device of the array

Here, ϕ_0^q is the velocity potential of the undisturbed incident harmonic wave; ϕ_7^q is the scattered potential around the q device, when it is considered fixed in waves with the duct open to the atmosphere, so that the pressure in the chamber is equal to the atmospheric one; ϕ_j^{qp} is the motion–dependent radiation potential around the body q resulting from the forced oscillation of the p–th body with unit velocity amplitude, $\dot{x}_j^p = \mathrm{Re}\left\{\dot{x}_{j0}^p \cdot e^{-i\omega t}\right\}$; ϕ_P^{qp} is the pressure–dependent radiation potential around the q–th body when it is considered fixed in the wave field and open to the atmosphere due to unit time harmonic oscillating pressure head, $P_{in}^p = \mathrm{Re}\left\{p_{in0}^p \cdot e^{-i\omega t}\right\}$, in the chamber of the p device which is considered fixed in otherwise calm water.

The velocity potential of the undisturbed incident wave system, ϕ_0^q, propagating at angle β, (Fig. 3), in the positive x–axis can be expressed in the cylindrical co–ordinate frame of the q–th body as follows [Mavrakos & Koumoutsakos, 1987]:

$$\phi_0^q(r_q,\theta_q,z) = -i\omega \frac{\mathrm{H}}{2} \sum_{m=-\infty}^{\infty} i^m \Psi_{0,m}^q(r_q,z) \cdot e^{im\theta_q} \tag{3}$$

where

$$\frac{1}{d}\Psi_{0,m}^q(r_q,z) = e^{ik\ell_{oq}\cos(\theta_{oq}-\beta)} \frac{Z_0(z)}{dZ_0'(d)} J_m(kr_q) e^{-im\beta} \tag{4}$$

The symbols used above are defined in Figure 3. Here J_m is the m–th order Bessel function of first kind and $Z_0(z)$ is defined by:

$$Z_0(z) = \left[0.5\left[1+\sinh(2kd)/(2kd)\right]\right]^{-1/2} \cdot \cosh(kz) \tag{5}$$

with $Z_0'(d)$ being its derivative at $z=d$. Frequency ω and wave number k are related by the dispersion equation.

The diffraction, i.e. $\phi_D^q = \phi_0^q + \phi_7^q$, the motion–, and pressure–dependent radiation potentials around the isolated q device, when it is considered alone in the field, are expressed in its own cylindrical co–ordinate system (r_q,θ_q,z) as follows:

$$\phi_D^q(r_q,\theta_q,z) = -i\omega \frac{H}{2} \sum_{m=-\infty}^{\infty} i^m \Psi_{D,m}^q(r_q,z) \cdot e^{im\theta_q} \tag{6}$$

$$\phi_j^{qq}(r_q,\theta_q,z) = -i\omega \sum_{m=-\infty}^{\infty} \Psi_{j,m}^{qq}(r_q,z) \cdot e^{im\theta_q} \tag{7}$$

$$\phi_P^{qq}(r_q,\theta_q,z) = \frac{1}{i\omega\rho} \sum_{m=-\infty}^{\infty} \Psi_{p,m}^{qq}(r_q,z) \cdot e^{im\theta_q} \tag{8}$$

Here ρ is the water density.

The potentials φ_j^l ($l \equiv q,\ qp$; j=D, 1, …, 6, P; p, q = 1, 2,…, N) are solutions of Laplace's equation in the entire fluid domain and satisfy the following boundary conditions:

$$\omega^2\phi_j^l - g\frac{\partial \phi_j^l}{\partial z} = \begin{cases} 0 & \text{for } r_q \ge a_q; \quad l \equiv q, j = D; \quad \text{or } \ l \equiv qp, j = 1,2,...,6,P \\ 0 & \text{for } b_{1,q} \le r_q \le b_q; \quad l \equiv q, j = D; \quad \text{or } \ l \equiv qp, j = 1,2,...,6 \\ -\delta_{q,p}\dfrac{i\omega}{\rho} & \text{for } b_{1,q} \le r_q \le b_q; \quad l \equiv qp; \quad j = P \end{cases} \tag{9}$$

at the outer and inner free sea surface $(z=d)$, and the zero normal velocity on the sea bed $(z=0)$. Furthermore, the potentials have to fulfil following kinematic conditions on the mean body's wetted surface:

$$\frac{\partial \varphi_j^l}{\partial \overrightarrow{n^q}} = 0, l \equiv q, j \equiv D; \text{or } l \equiv qp, j \equiv P \tag{10a}$$

$$\frac{\partial \varphi_j^{qp}}{\partial \overrightarrow{n^q}} = \delta_{q,p} n_j^p \quad \text{for} \quad j = 1,..,6 \tag{10b}$$

Here $\partial(\ )/\partial\overrightarrow{n^q}$ denotes the derivative in the direction of the outward unit normal vector $\overrightarrow{n^q}$, to the mean wetted surface S_0^q on the q–th body, and n^p, are its generalized components defined as $\overrightarrow{n^p} = (n_1^p, n_2^p, n_3^p)$ and $\overrightarrow{r^p} \times \overrightarrow{n^p} = (n_4^p, n_5^p, n_6^p)$, where $\overrightarrow{r^p}$ is the position vector with respect to the origin of the coordinate system. Finally, a radiation condition must be imposed which states that propagating disturbances must be outgoing.

The unknown potential functions $\Psi_{j,m}^{k,l}$, k=I, III, M, IV, see Eqs. (6)–(8) can be established in each fluid region surrounding the q–th device through the method of matched axisymmetric eigenfunction expansions.

The potentials, ϕ_j^{qp}, (j=1, …, 6, P) around anybody q of the configuration due to oscillation of body p in otherwise calm water (motion – dependent radiation potential) or due to inner time harmonic oscillating pressure head in the chamber of body p (pressure – dependent radiation potential), can be expressed in the q–th body's cylindrical coordinate system, as:

$$\phi_j^{qp}(r_q,\theta_q,z) = -i\omega \sum_{m=-\infty}^{\infty} \Psi_{j,m}^{qp}(r_q,z) \cdot e^{im\theta_q} \tag{11}$$

$$\varphi_p^{qp}(r_q,\theta_q,z) = \frac{1}{i\omega\rho} \sum_{m=-\infty}^{\infty} \Psi_{p,m}^{qp}(r_q,z) \cdot e^{im\theta_q} \tag{12}$$

In order to express the potentials, ϕ_j^{qp}, in the form of Eqs. (11) and (12), use is made of the multiple scattering approach (Twersky, 1952; Okhusu, 1974). This method has been further elaborated to solve the diffraction and the motion – dependent radiation problems around arbitrarily shaped, floating or / and submerged vertical axisymmetric bodies by Mavrakos & Koumoutsakos (1987) and Mavrakos (1991) and for the diffraction and the pressure – dependent radiation problems for an interacting array of OWC's devices by Konispoliatis & Mavrakos (2013b); thus, it will be no further elaborated here.

VOLUME FLOW

The time dependent volume flow produced by the oscillating internal water surface in q OWC device, q=1,2,…,N, is denoted by $Q^q(r_q,\theta_q,z;t) = \mathrm{Re}\left[q^q(r_q,\theta_q,z) \cdot e^{-i\omega t}\right]$, where:

$$q^q = \iint_{S_i^q} u_z dS_i = \iint_{S_i^q} u_z(r_q,\theta_q,z=d) r_q dr_q d\theta_q = \iint_{S_i^q} \frac{\partial \phi^q}{\partial z} r_q dr_q d\theta_q \tag{13}$$

Here u_z denotes the vertical velocity of the water surface, and S_i^q the inner water surface in the q device.

Assuming that the Wells turbine is placed in a duct between the q device's chamber and the outer atmosphere and that it is characterized by a pneumatic admittance Λ^q, then the total volume flow is equal to [Evans & Porter; 1996, Falnes; 2002]:

$$Q^q(t) = \Lambda^q \cdot P_{in}^q(t) \tag{14}$$

According to [Sarmento & Falcao; 1985, Falcao 2002], for the Wells turbine, can be obtained:

$$q^q = \left[\frac{KD}{\rho_a N} + (-i\omega)\frac{V_0}{\gamma P_a}\right] \cdot p_{in0}^q \tag{15}$$

Where K is constant for a given turbine geometry (independent of turbine size or rotational speed), D is turbine rotor diameter, N is the rotational speed (radians per unit time) and ρ_a, P_a are the atmospheric density and pressure.

Decomposing the total volume flow, q^q, of the q–th device, same as for the velocity potential; see Equation (2), into three terms associated with the diffraction, q_D^q, and the motion– and pressure– dependent radiation problems, q_R^q, q_p^q, respectively, we can obtained:

$$q^q = q_D^q + q_R^q + \sum_{p=1}^{N} p_{in0}^p \cdot q_P^{qp} \tag{16}$$

Here: $q_R^q = \sum\limits_{p=1}^{N}\sum\limits_{j=1}^{6} \dot{x}_{j0}^p \cdot q_{3,j}^p - \dot{x}_{30}^q S_i^q$ (17)

After substituting Equation (15) into Equation (16), it reveals convenient to express (16) in matrix formulation, for all the devices of the array, i.e.:

$$\lfloor \lambda^N \rfloor \cdot \lfloor x_{j0}^N \rfloor + \lfloor B^N \rfloor \cdot \lfloor p_{in0}^N \rfloor = \lfloor q_D^N \rfloor \tag{18}$$

Here: $\lfloor q_D^N \rfloor, \lfloor p_{in0}^N \rfloor$ are (Nx1) vectors containing the diffraction volume flows and the inner oscillating pressure head, respectively, in all the devices of the array; $\lfloor B^N \rfloor$ is a (N x N) square matrix; $\lfloor x_{j0}^N \rfloor$ is a (6Nx1) vector containing the prescribed motion displacements of each device of the array and $\lfloor \lambda^N \rfloor$ is a (Nx6N) matrix.

The elements of the (N x N) square matrix, $\lfloor B^N \rfloor$, and (Nx6N) matrix, $\lfloor \lambda^N \rfloor$, are $\delta_{q,p}\left(\frac{KD}{\rho_a N} + (-i\omega)\frac{V_0}{\gamma P_a}\right) - q_P^{qp}$ and $i\omega q_{3,j}^{qp} - i\omega S_i^q \delta_{q,p}\delta_{3,j}$, respectively, for every device of the array.

HYDRODYNAMIC FORCES

The various forces on the q oscillating water column device can be calculated from the pressure distribution given by the linearised Bernoulli's equation:

$$P(r_q, \theta_q, z; t) = -\rho \frac{\partial \Phi^q}{\partial t} = i\omega\rho\phi^q \cdot e^{-i\omega t} \tag{19}$$

Where ϕ^q is the q devices' velocity potential in each fluid domain I, III, M and IV. The horizontal and vertical exciting forces and moments acting on an array of OWC devices have been presented in Konispoliatis & Mavrakos (2013b).

The hydrodynamic reaction forces and moments F_{ij}^{qp} acting on the device q in the i–th direction due to the oscillation of body p in the j–th direction, can be calculated by the Equation (19) and the complex form f_{ij}^{qp} may be written in the form (Newman, 1977):

$$f_{ij}^{qp} = \omega^2\left(a_{ij}^{qp} + \frac{i}{\omega} b_{ij}^{qp}\right) \cdot x_{j0}^p = \omega^2 \pi_{ij}^{qp} . x_{j0}^p \tag{20}$$

Here, a_{ij}^{qp}, b_{ij}^{qp}, are the well–known added mass and damping coefficients.

In the same way, the hydrodynamic pressure forces and moments f_i^{qp} acting on the device q in the i–th direction due to oscillating pressure head in the p device can be written in the form:

$$f_i^{qp} = \left(-e_i^{qp} + i d_i^{qp}\right) \cdot p_{in0}^p \tag{21}$$

Here e_i^{qp}, d_i^{qp} are the pressure added mass and dumping coefficients, respectively.

The total hydrodynamic forces on the entire multi–body configuration when it is considered as a unit can be calculated by properly superposed the corresponding forces on each device with respect to the reference point of motion, G, of the entire structure. To this end use is made of the following 6x6 square matrix which contains the coordinates of the reference point for the motions of the p–th device with respect to the reference point for the motions of the entire configuration (for details see Mavrakos, 1991), i.e.:

$$\lfloor B^p \rfloor = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -z^p & y^p & 1 & 0 & 0 \\ z^p & 0 & -x^p & 0 & 1 & 0 \\ -y^p & x^p & 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

Here (x^p, y^p, z^p) are the coordinates of the reference point for the motions of the p–th device with respect to the global co – ordinate system G.

MOTION – PRESSURE EQUATIONS

The investigation of the equilibrium of the forces acting on the freely floating array of OWC devices leads to the following differential system of motion equations, in the frequency domain, i.e.:

$$\sum_{p=1}^{N}\sum_{j=1}^{6}(\delta_{p,q} m_{kj}^q + a_{kj}^{qp}) \cdot \ddot{x}_{j0}^p + b_{kj}^{qp} \cdot \dot{x}_{j0}^p + \delta_{p,q} c_{kj}^q \cdot x_{j0}^p = f_k^q, \quad k = 1,\ldots,6 \tag{23}$$

where m_{kj}^q and c_{kj}^q are elements of the (6x6) mass and stiffness matrices, respectively, and f_k^q is the sum of the exciting and pressure hydrodynamic forces acting on the device q at the k–th direction.

Next, by accounting for the Eq. (21), Eq. (23) can be written in the following matrix form:

$$\lfloor \mu^N \rfloor \cdot \lfloor x_{j0}^N \rfloor + \lfloor C^N \rfloor \cdot \lfloor p_{in0}^N \rfloor = \lfloor f_D^N \rfloor \qquad (24)$$

Here $\lfloor \mu^N \rfloor$ is a (6*N*x6*N*) square matrix; $\lfloor C^N \rfloor$ is a (6*N*x*N*) matrix and $\lfloor f_D^N \rfloor$ is (6*N*x1) vector containing the exciting forces acting on each device of the array.

The elements of the (6*N*x6*N*) square matrix, $\lfloor \mu^N \rfloor$, and (6*N*x*N*) matrix, $\lfloor C^N \rfloor$, are $\left(-\omega^2 \cdot \left[\delta_{p,q} m_{kj}^q + a_{kj}^{qp} + \frac{i}{\omega} b_{kj}^{qp}\right] + \delta_{p,q} c_{kj}^q\right)$, and $\left(-f_i^{qp} / p_{in0}^p - \delta_{p,q} S_i^q\right)$, respectively, for every device of the array.

The motion components for each freely floating device of the array and the pressure in each device's chamber are the unknown terms of the problem. These terms can be obtained as the solution of the Equations (18) and (24).

When the multi–body configuration is considered as a unit, then the system of motion equations can be written as:

$$\sum_{j=1}^{6} \left[-\omega^2 (M_{i,j} + A_{i,j} + \frac{i}{\omega} B_{i,j}) + C_{i,j} \right] \cdot x_{j0} = F_i, \quad i = 1,2...,6 \qquad (25)$$

where $M_{i,j}$ and $C_{i,j}$ are elements of the (6x6) mass and stiffness matrices of the entire configuration; $A_{i,j}, B_{i,j}$, are the hydrodynamic masses and potential damping of the entire configuration; F_i is the sum of the exciting and pressure hydrodynamic forces acting on the multi–body system at the i–th direction and x_{j0} is the motion displacement of the entire OWC system at the i–th direction with respect to a global co – ordinate system G.

The linear translation and rotational motions of the entire OWC system, x_{j0}, can be expressed though the translational and angular motions of the p–th device, x_{j0}^p, using the following relations

$$x_{10}^p = x_{10} + x_{50} z^p - x_{60} y^p, \; x_{20}^p = x_{20} - x_{40} z^p + x_{60} x^p, \qquad (26)$$
$$x_{30}^p = x_{30} + x_{40} y^p - x_{50} x^p, \; x_{40}^p = x_{40}, \; x_{50}^p = x_{50}, \; x_{60}^p = x_{60}$$

Here, (x^p, y^p, z^p) are the coordinates of the reference point for the motions of the p–th device, with respect to G. In this way, the displacement of the entire OWC system and the inner pressure head inside each of the devices when they assumed as a rigid platform can be calculated as an extension of the differential systems of the Equations (18) and (24).

POWER ABSORPTION

The power absorbed, P, by each OWC device of the array is the sum of the power absorbed by the q–th device as a result of oscillation in mode j, and the power absorbed through the oscillating internal water surface (Falnes, 2002), i.e.:

$$P = \frac{1}{2} \mathrm{Re}\left\{ \sum_{j=1}^{6} \overline{f_j^q} \cdot \dot{x}_{j0}^q \right\} + \frac{1}{2} \mathrm{Re}\left\{ \overline{p_{in0}^q} \cdot q^q \right\} \qquad (27)$$

Here, $\overline{p_{in0}^q}, \overline{f_j^q}$ are the complex conjugates of p_{in0}^q, f_j^q, respectively. The terms f_j^q and q^q in a matrix form, from Equations (24) and (18), can be written as:

$$\lfloor f^q \rfloor = \lfloor f_D^N \rfloor + \lfloor \mu^{*N} \rfloor \cdot \lfloor \dot{x}_{j0}^N \rfloor + \lfloor C^{*N} \rfloor \cdot \lfloor p_{in0}^N \rfloor \qquad (28)$$

$$\lfloor q^q \rfloor = \lfloor q_D^N \rfloor + \lfloor \lambda^{*N} \rfloor \cdot \lfloor \dot{x}_{j0}^N \rfloor + \lfloor B^{*N} \rfloor \cdot \lfloor p_{in0}^N \rfloor \qquad (29)$$

where $\lfloor f^q \rfloor$, $\lfloor q^q \rfloor$ are (6*N*x1), (*N*x1), vectors containing the f_j^q and q^q elements, respectively; $\lfloor \dot{x}_{j0}^N \rfloor$ is a (6*N*x1) vector containing the velocity amplitude of each device of the array; $\lfloor C^{*N} \rfloor = -\lfloor C^N \rfloor$, see Equation (24), and $\lfloor f_D^N \rfloor$, $\lfloor q_D^N \rfloor$ matrices are defined at Equations (24), (18), respectively.

The elements of the (*N*x*N*) square matrix, $\lfloor B^{*N} \rfloor$, are q_P^{qp}; the elements of the (6*N*x6*N*) square matrix, $\lfloor \mu^{*N} \rfloor$, are $f_{ij}^{qp} / \dot{x}_{j0}^p$ and the elements of the (*N*x6*N*) matrix, $\lfloor \lambda^{*N} \rfloor$, are $q_{3,j}^{qp} - S_i^q \delta_{q,p} \delta_{3,j}$.

The Equation (27), through Equations (28) and (29) can be written in a matrix form (Falnes, 2002), i.e.:

$$P = \frac{1}{4} \left\{ \kappa^{N^T} \cdot \overline{u^N} + \overline{\kappa^{N^T}} \cdot u^N \right\} + \frac{1}{2} \cdot \overline{u^{N^T}} \cdot \Delta^N \cdot u^N \qquad (30)$$

where κ^N, u^N, Δ^N are complex vectors, (7*N*x1), and complex hermitian matrix, (7*N*x7*N*), respectively, equal to:

$$\kappa^N = \begin{bmatrix} \lfloor f_D^N \rfloor \\ \lfloor q_D^N \rfloor \end{bmatrix}, \; u^N = \begin{bmatrix} \lfloor \dot{x}_{j0}^N \rfloor \\ \lfloor p_{in0}^N \rfloor \end{bmatrix}, \; \Delta^N = \begin{bmatrix} \mathrm{Re}\lfloor \mu^{*N} \rfloor & i \cdot \mathrm{Im}\lfloor C^{*N} \rfloor \\ -i \cdot \mathrm{Im}\lfloor C^{*N} \rfloor^T & \mathrm{Re}\lfloor B^{*N} \rfloor \end{bmatrix} \qquad (31)$$

Here "*T*" denotes the transpose matrix.

If there is no constraint on the complex amplitudes of the components of column vector u^N, the maximum value of the absorbed power is (Falnes, 2002):

$$P_{\max} = \frac{1}{8} \left\{ \overline{\kappa^{N^T}} \cdot E^{N^{-1}} \cdot \kappa^N \right\} \qquad (32)$$

corresponding to an optimum oscillator amplitude vector

$$u_{opt}^N = \frac{1}{2} \left\{ E^{N^{-1}} \cdot \kappa^N \right\} \qquad (33)$$

where, $E^{N^{-1}}$ is the inverse matrix of $E^N = -\Delta^N$.

NUMERICAL MODELLING

A numerical model of an array of three identical OWC devices placed at a triangle ordinance (Figure 4) was built in the frequency domain. Each device's oscillating chamber was supplemented by a concentric interior piston– like truncated cylinder. Three configurations were examined; firstly, the examined devices were assumed to float independently; secondly, they were connected together forming a freely floating multi–device system and finally, a mooring system was installed to the system of OWC devices. The presented results were obtained using the Hydrodynamic Analysis of Multiple Vertical Axisymmetric Bodies (HAMVAB) software.

The geometrical characteristics of each of the OWC devices are: $a_q \approx b_q$; $b_{1,q} = 0.357 \cdot b_q$; $d = 14.29 \cdot b_q$; $d - h_q = (4/7) \cdot b_q$; $d - h_{1,q} = (10/7) \cdot b_q$ and $\ell_{12} = \ell_{23} = \ell_{13} = 3.57 \cdot b_q$; $b_q = 14m$ (for definitions see Figs. 3,4). For the first configuration the mass and the mass moment of inertia relative to the reference point of motions, is $1646.195\,tn$ and $17169\,tn.m^2$, respectively. For the second configuration the mass and the mass moment of inertia of the entire multi–body system considered as a unit with respect to the centre of gravity, is $4938{,}6\,tn$ and $I_{xx} = I_{yy} = 1106000\,tn.m^2$, $I_{zz} = 1987000\,tn.m^2$, respectively. The coordinates of the centre of gravity are (28.86, 0,-4.05). In the third configuration the platform is moored through vertical tendons as a Tension Leg Platform (TLP). The mass of the OWC's system is assumed 1635,7*tn* and the mass moment of inertia of the entire multi–body system with respect to the centre of gravity, around X-,Y-,Z- axis, are the same as the above. Each mooring line attachment point is located at the vertical axis of each interior concentric cylinder. The point's z–coordinate on each device, relatively to the global co–ordinate system *G* is 180*m*. The spring constant of each of the mooring lines in the x and y direction due to unit translational motions

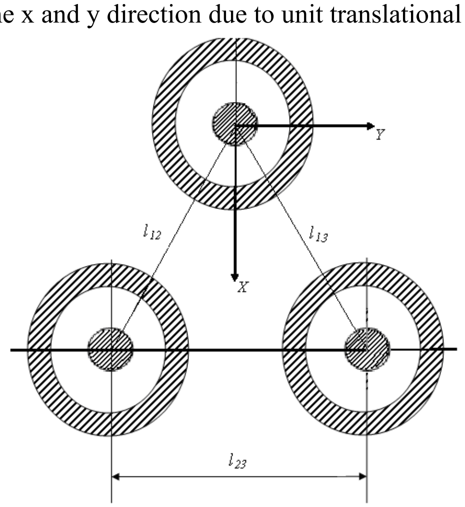

Figure 4: Array of three oscillating water column devices placed at a triangle ordinance

of each attachment point along these directions is $60\,kN/m$, whereas in the z direction is $14700\,kN/m$. Finally, the pretension of each of the mooring lines in the z direction is $10800\,kN$.

A regular monochromatic wave is propagating along the positive x–axis at zero angle of incidence, the origin of the global cartesian co–ordinate system is on the sea bed and its vertical axis directed positive upward coinciding with the vertical axis of the first device local coordinate system (Figure 4).

The pneumatic admittance Λ^q for all the OWC's was considered as a real and positive number equal to the optimum coefficient Λ_{opt} of the same restrained OWC device but in isolation condition as in Evans and Porter (1996) work.

Before presenting the results obtained from the model, we shall introduce the non–dimensional form of the parameters of interest.

The non–dimensional exciting forces on each device *q* of the array and on the entire OWC system, $\tilde{f}_i^q, \tilde{f}_i^T, i = x,z$, respectively, are defined as:

$$\left(\tilde{f}_x^q, \tilde{f}_z^q\right) = \left(\frac{f_x^q}{\rho g a_q^2 (H/2)}, \frac{f_z^q}{\rho g a_q^2 (H/2)}\right), \; q = 1,2,3 \tag{34}$$

$$\left(\tilde{f}_x^T, \tilde{f}_z^T\right) = \left(\frac{f_x^T}{3\rho g a_q^2 (H/2)}, \frac{f_z^T}{3\rho g a_q^2 (H/2)}\right) \tag{35}$$

The non–dimensional inner air pressure in each device *q* is defined as:

$$\tilde{p}_{in0}^q = p_{in0}^q \big/ \rho g (H/2), \; q = 1,2,3 \tag{36}$$

Here *g* is the acceleration of gravity.

The non–dimensional horizontal– vertical– displacement and the pitch rotation of each device q, $\left(\tilde{x}_{10}^q, \tilde{x}_{30}^q, \tilde{x}_{50}^q\right)$ and of the multi–device system, $\left(\tilde{x}_{10}^T, \tilde{x}_{30}^T, \tilde{x}_{50}^T\right)$, respectively, is defined by:

$$\left(\tilde{x}_{10}^q, \tilde{x}_{30}^q, \tilde{x}_{50}^q\right) = \left(x_{10}^q \big/ (H/2), x_{30}^q \big/ (H/2), x_{50}^q \big/ (k \cdot H/2)\right) \tag{37}$$

$$\left(\tilde{x}_{10}^T, \tilde{x}_{30}^T, \tilde{x}_{50}^T\right) = \left(x_{10} \big/ (H/2), x_{30} \big/ (H/2), x_{50} \big/ (k \cdot H/2)\right) \tag{38}$$

The non–dimensional absorbed power from each device *q* is defined as:

$$E^q = P^q \big/ \omega \rho g a_q^2 (H/2)^2, \; q = 1,2,3 \tag{39}$$

In Figures 5 and 6 the horizontal, $\tilde{f}_x^q$, $\tilde{f}_x^T$, and vertical, $\tilde{f}_z^q$, $\tilde{f}_z^T$, exciting forces acting on each device *q* of the array, and on the multi–device system are plotted versus ω, *q*=1,2,3.

In Figures 7, 8, 9 the horizontal and vertical motions and the pitch angle are plotted versus ω. In these figures the displacement and rotation of each freely floating body, $\left(\tilde{x}_{10}^{q}, \tilde{x}_{30}^{q}, \tilde{x}_{50}^{q}\right)$, and the displacement and rotation of the freely floating or moored multi–body array, $\left(\tilde{x}_{10}^{T}, \tilde{x}_{30}^{T}, \tilde{x}_{50}^{T}\right)$, is presented. It is noticeable that the vertical displacement of the moored multi–body array is much lower than the vertical displacement of the individual floating OWC's and of the freely floating multi–body array. Thus the air pressure inside each device of the moored multi–body system would be higher than in the rest configurations.

A better view of the above conclusion can be seen at Figures 10 and 11 where the modulus of the inner pressure, $\tilde{p}_{in0}^{q}$, q=1,2,3, in each device of the array when they are floating separately; or as a unit; or they are moored as a unit versus ω, is being presented. It is noticeable that the air pressure inside the devices of the moored multi–body array is much higher than in the devices floating independently or as a

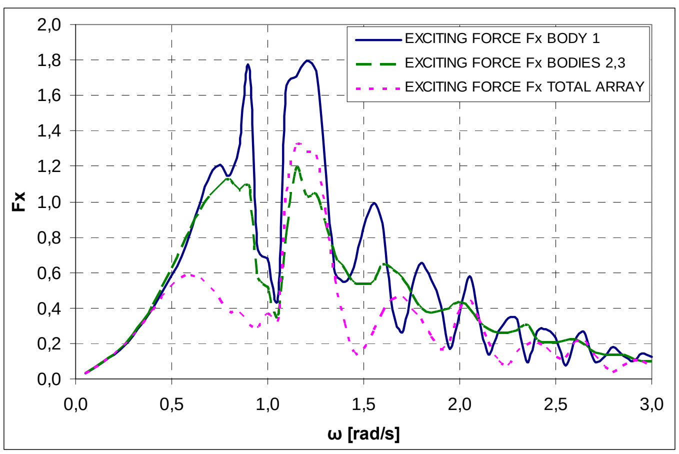

Figure 5: Horizontal exciting forces through x axis acting on each device of the array and on the multi–body configuration

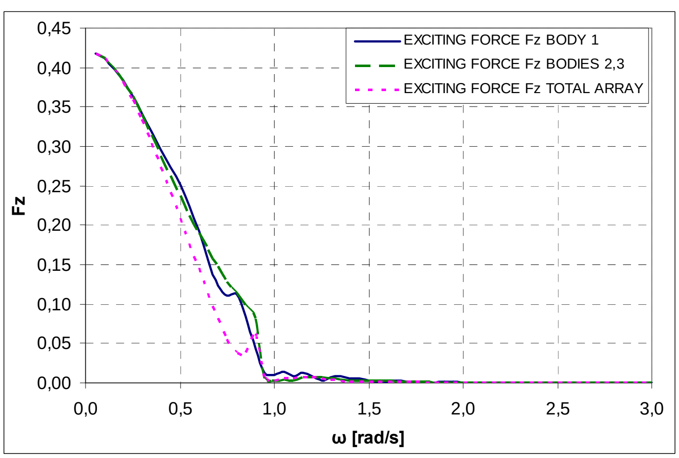

Figure 6: Vertical exciting forces acting on each device of the array and on the multi–body configuration

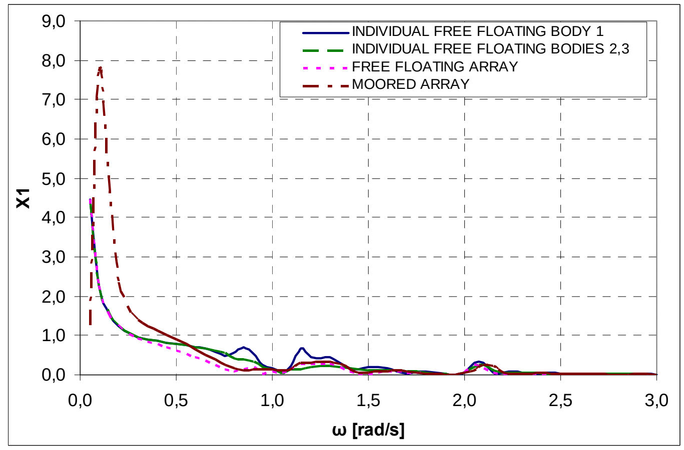

Figure 7: Horizontal displacement (surge motion) of each freely floating OWC and of the freely floating or moored multi–body array

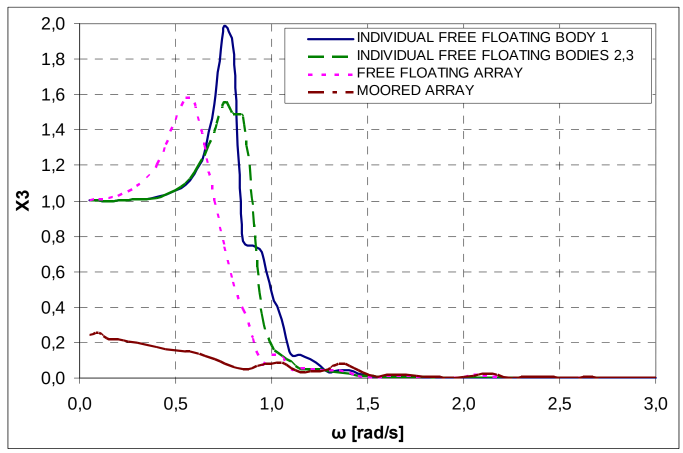

Figure 8: Vertical displacement (heave motion) of each freely floating OWC and of the freely floating or moored multi–body array

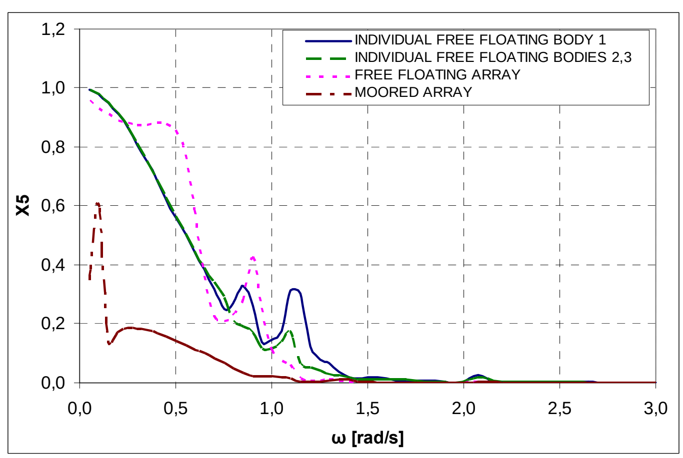

Figure 9: Pitching angle of each freely floating OWC and of the freely floating or moored multi–body array

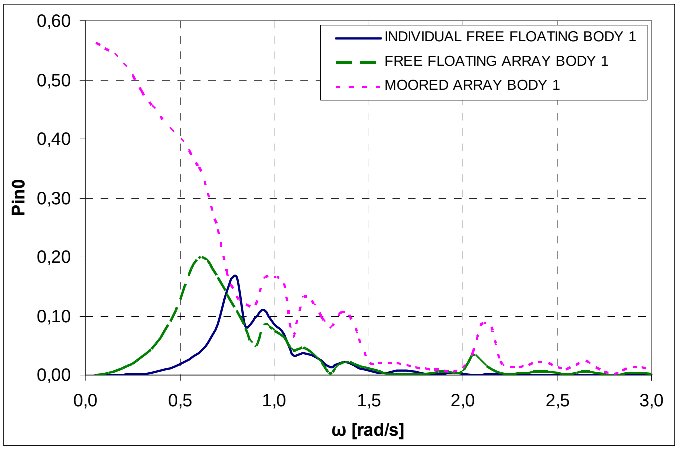

Figure 10: Modulus of the inner pressure inside the first device when it is floating independently; as member of the floating unit; as member of the moored unit

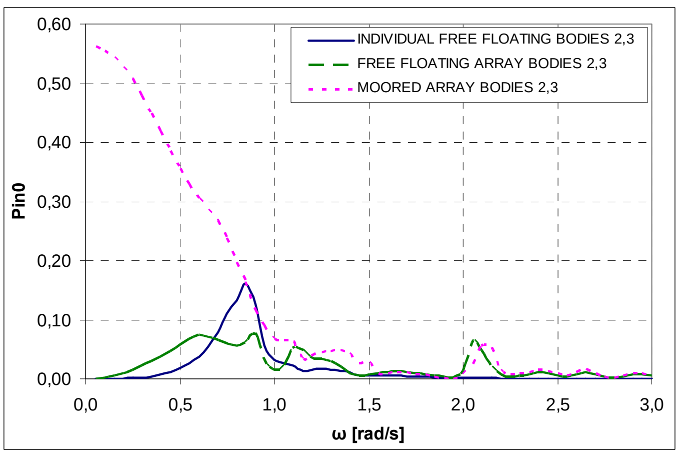

Figure 11: Modulus of the inner pressure inside the second – third device when it is floating independently; as member of the floating unit; as member of the moored unit

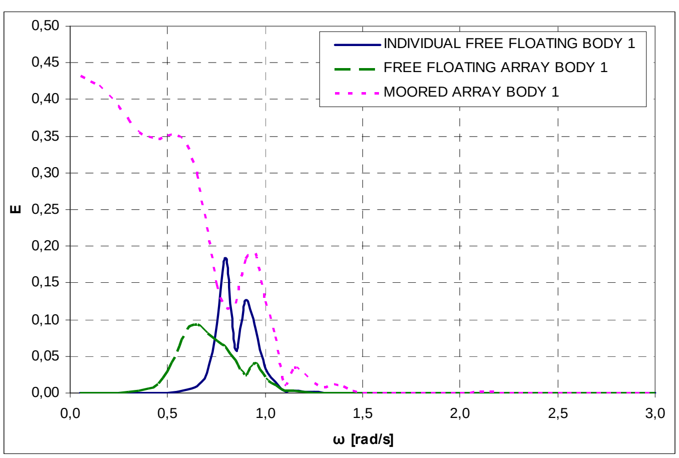

Figure 12: Absorbed wave energy by the first OWC device of the array when it is floating independently; as member of the floating unit; as member of the moored unit

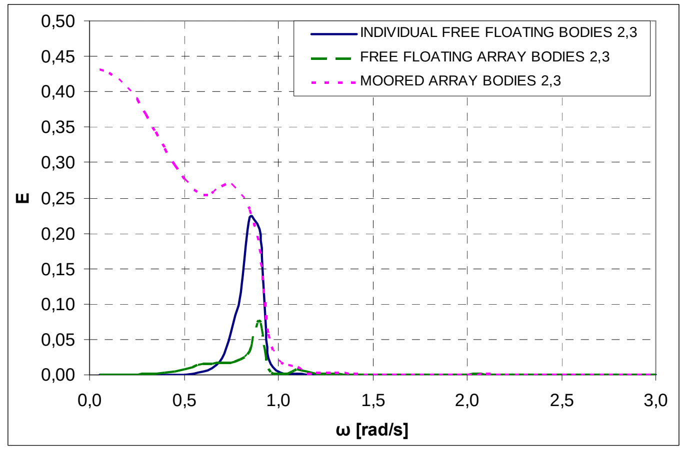

Figure 13: Absorbed wave energy by the second – third OWC device of the array when it is floating separately; as a unit; moored as a unit with the others

unit. It is reminded that in order to produce the inner air pressure results we claimed that the turbine's pneumatic admittance is equal to an optimum value which maximizes the absorbed energy by the OWC device when it is considered restrained in wave impact and in isolation condition.

Finally, at Figures 12 and 13 the absorbed energy, E^q , by each of the three devices when they are floating separately; or as a unit; or they are moored as a unit, versus ω is depicted. As it can be seen from the figures the devices of the moored multi–body array absorb more wave energy than the floating independently or as a unit OWC's.

CONCLUSIONS

An exact method for solving the diffraction, the motion– and the pressure– dependent radiation problems around an array of floating OWC devices has been presented. The produced inner pressure in each OWC device and the motion components of each OWC were obtained by solving a differential motion–pressure equation system. The pressure distributions inside the chambers for the case of independently freely floating devices, as well as for the case of freely floating and moored array of OWC's were presented and compared between each other. The results of this analysis show the importance of the hydrodynamic interaction effects and the motion characteristics of each device in evaluating the absorbed wave power.

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