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DRAFT: HYDRODYNAMICS OF MULTIPLE VERTICAL AXISYMMETRIC OWC'S DEVICES RESTRAINED IN WAVES

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ABSTRACT

An increased interest in the use of Oscillating Water Column (OWC) devices in recent years has led to the consideration of a number of differentiations in geometry (wall thickness, draught, shape of the chamber), topography installation (on-, near-, off - shore), moving conditions (restrained, freely floating bodies) and number of used devices in order to extract maximum wave energy. Many considerable efforts and advances have been made to wave power absorption by isolated devices; however, much less attention has been paid to the influence of neighbouring OWC structures on wave loading and wave energy extraction. This paper deals with the linearised hydrodynamic interaction problem of regular, small amplitude, surface gravity waves and a stationary group of vertical axisymmetric OWC devices.

1 INTRODUCTION

The problem of the hydrodynamic interaction among neighbouring OWC devices is of a particular importance in evaluating the absorbed wave energy by a device since the hydrodynamic characteristics of each member of a multi-body configuration may differ from the ones obtained for an isolated device due to hydrodynamic interaction phenomena. Each device of the configuration scatters waves towards the others, which in turn scatter waves contributing to the excitation of the initial device and so on. Here, the method of multiple scattering will be used to capture the hydrodynamic interaction phenomena among the multiple – body arrangement. The method relies on single OWC device hydrodynamic characteristics and takes into account the interaction effects by the implementation the physical idea of multiple scattering. The latter was introduced by Twersky (1952) in studying the acoustic scattering by an array of parallel cylinders and was applied to free-surface wave interactions with floating bodies by Ohkusu (1974) who investigated the case of three adjacent, floating, vertical truncated cylinders. The method was extended by Mavrakos and Koumoutsakos (1987) and Mavrakos (1991) for the solution of the diffraction and radiation problems by an array of arbitrarily shaped vertical axisymmetric bodies with any geometrical arrangement and individual bodies' geometries. For the isolated body – wave interaction, the method of matched eigenfunction expansions is used for the evaluation of the diffraction (body fixed in wave, atmospheric pressure in the chamber) and the radiation velocity potentials in properly defined fluid domains surrounding each device. The radiation potential results from an oscillating pressure head acting on the inner free surface of the OWC.

In the present contribution, numerical results concerning exciting wave forces and moments on each device of the OWC array, as well as air flow rates associated with the diffraction problem in each body are presented. In addition, forces originated from the air pressure head in each device's chamber and air flow rates due to pressure-dependent-radiation problem, for several turbine parameters are computed. The value of the inner pressure head in each device of the OWC array is also compared with the corresponding one of the isolated OWC device and the influence of neighbouring bodies on the total wave field and the associated hydrodynamic loading of the individual columns assessed.

2 FORMULATION OF THE HYDRODYNAMIC PROBLEM

We consider a stationary group of N rigid vertical axisymmetric oscillating water column devices excited by a plane periodic wave of amplitude $H/2$, frequency ω and wave number k propagating in water of finite water depth d. The outer and inner radii of each device q, q=1, 2,…, N, are a_q, b_q, respectively, whereas the distance between the bottom of the q device and sea bed is denoted by h_q (Fig. 1). It is assumed small amplitude, inviscid, incompressible and irrotational flow, so that linear potential theory can be employed. A global Cartiesian co–ordinate system O–XYZ with origin on the sea bed and its vertical axis OZ directed positive upwards is used. Moreover, N local cylindrical co-ordinate systems $\left(r_q, \theta_q, z_q\right)$, q = 1, 2,…, N are defined with origins on the sea bottom and

their vertical axes pointing upwards and coinciding with the vertical axis of symmetry of the q device.
The fluid flow around the q=1,2,…,N device can be described by the potential function:

$$\Phi^q\left(r_q,\theta_q,z;t\right)=\mathrm{Re}\left\{\phi^q\left(r_q,\theta_q,z\right)\cdot e^{-i\omega t}\right\} \qquad (1)$$

Following Evans (1982) the spatial function ϕ^q can be decomposed, on the basis of linear modelling, as:

$$\varphi^q=\varphi_0^q+\varphi_7^q+\sum_{p=1}^{N}\varphi_p^{qp}\left(r_q,\theta_q,z_q\right) \qquad (2)$$

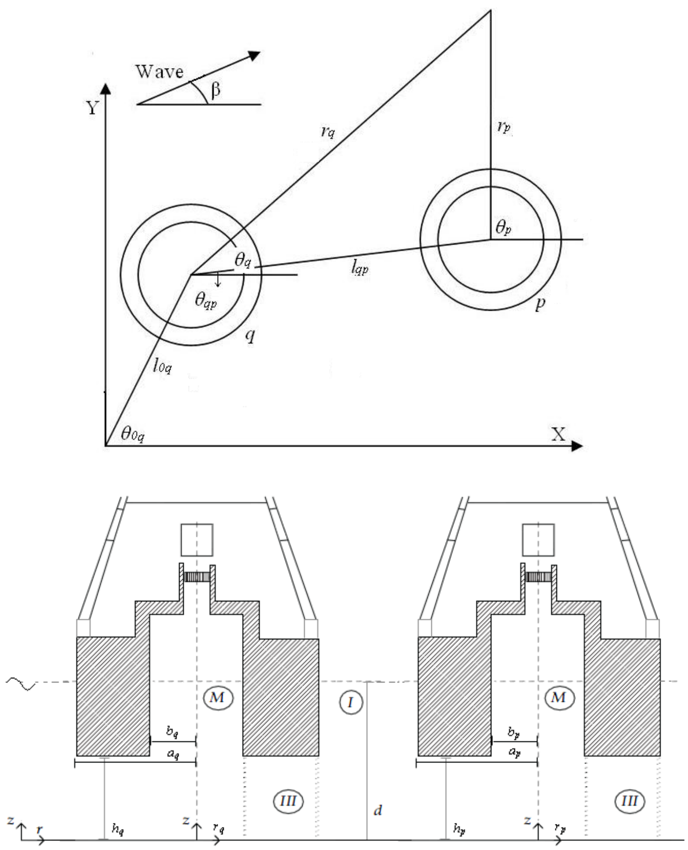

Figure 1. Definition sketch

Here, ϕ_0^q is the velocity potential of the undisturbed incident harmonic wave; ϕ_7^q is the scattered potential around the q device, when it considered fixed in waves with the duct open to the atmosphere, so that the pressure in the chamber is equal to the atmospheric one; ϕ_p^{qp} is the radiation potential around the q-th body due to time harmonic oscillating pressure head, $P_{in}^p=\mathrm{Re}\left\{p_{in0}^p\cdot e^{-i\omega t}\right\}$, in the chamber for the p device which is considered fixed in otherwise calm water.
The potentials φ_j^l ($l\equiv q,\ qp$; j=0, 7, P; p=1, 2,…, N) are solutions of Laplace's equation in the entire fluid domain and satisfy in the entire fluid domain the following boundary conditions:

$$\omega^2\varphi_j^l-g\frac{\partial\varphi_j^l}{\partial z}=\begin{cases}0 & \text{for}\quad r_q\geq a_q;l\equiv q\text{ or }qp \quad j=0,7,P\\ 0 & \text{for}\quad 0\leq r_q\leq b_q, \quad j=0,7\\ -\delta_{qp}\dfrac{i\omega}{\rho}p_{in0}^q & \text{for}\quad 0\leq r_q\leq b_q;l\equiv qp \quad j=P\end{cases} \qquad (3)$$

at the outer and inner free sea surface $\left(z=d\right)$,

$$\frac{\partial\phi_j^q}{\partial z}=0 \quad \text{for} \quad j=7,P \;\text{ on the sea bed }\left(z=0\right) \qquad (4)$$

$$\frac{\partial\phi_j^q}{\partial\vec{n}}=0 \quad \text{for} \quad j=7,P \qquad (5)$$

on the mean device's wetted surface S_0^q

Where $\partial(\ )/\partial\vec{n}$ denotes the derivative in the direction of the outward unit normal vector $\vec{n}$, to the mean wetted surface S_0^q on the q–th body. Finally, a radiation condition must be imposed which states that propagating disturbances must be outgoing.
The velocity potential of the undisturbed incident wave system, ϕ_0^q, propagating at angle β, (Fig. 1), to the positive x–axis can be expressed in the cylindrical co–ordinate frame of the q–th body as follows [Mavrakos & Koumoutsakos, 1987]:

$$\phi_0^q(r_q,\theta_q,z)=-i\omega\frac{H}{2}\sum_{m=-\infty}^{\infty}i^m\Psi_{0,m}(r_q,z)e^{im\theta_q} \qquad (6)$$

Where

$$\frac{1}{d}\Psi_{0,m}(r_q,z)=e^{ik\ell_{oq}\cos(\theta_{oq}-\beta)}\frac{Z_0(z)}{dZ_0'(d)}J_m(kr_q)e^{-im\beta} \qquad (7)$$

In accordance with the above Equation (6) the scattered and the pressure radiation potential due to the oscillating pressure head in the q-th device, expressed in the isolated q–th device's cylindrical co–ordinate system $\left(r_q,\theta_q,z\right)$ can be described by:

$$\phi_7^q(r_q,\theta_q,z)=-i\omega\frac{H}{2}\sum_{m=-\infty}^{\infty}i^m\Psi_{7,m}^q(r_q,z)e^{im\theta_q} \qquad (8)$$

$$\varphi_P^{qq}(r_q,\theta_q,z)=\frac{p_{in0}^q}{i\omega\rho}\sum_{m=-\infty}^{\infty}\Psi_{P,m}^{qq}(r_q,z)e^{im\theta_q} \qquad (9)$$

The unknown potential functions $\Psi_{j,m}^q$, and $\Psi_{P,m}^{qq}$ $j=7,P$, involved in (8), (9) can be established through the method of matched axisymmetric eigenfunction expansions.
According to this method, the flow field around the device q is subdivided in coaxial ring–shaped fluid regions, categorized by the numerals I, III and M (Fig. 1). In each fluid domain, different series expansions of the velocity potential are made. The adopted series representations, which are solutions of the Laplace equation in each fluid region, are selected in such a way' that satisfy the corresponding conditions at the horizontal boundaries of each fluid region and, in addition, the radiation condition at infinity in the outer fluid domain. As a result, the

velocity potentials in each fluid domain fulfil a priori the kinematical boundary conditions at the horizontal walls of the bottomless cylindrical duct, the linearized condition at the free surface, the kinematical one on the sea bed, and the radiation condition at infinity.

At this point it should be mentioned that although the radiation potential φ_P^{qq}, around the isolated body q involves only the $m = 0$ term [Mavrakos & Konispoliatis, 2012], its series representations in form of Eq. 9 has been preferred at this stage of the analysis in order to obtain a similar representation with the one of the total potential, ϕ_P^{qp}, induced around any body q of the configuration due to the inner pressure in body p. This potential has to be expressed in the q–th body's cylindrical coordinate system (r_q, θ_q, z) and generally includes components for all values of m accounting for interference effects. Indeed, in accordance with Equation (9), ϕ_P^{qp} can be expressed as:

$$\phi_P^{qp}(r_q, \theta_q, z) = \frac{p_{in0}^{p}}{i\omega\rho} \sum_{m=-\infty}^{\infty} \Psi_{P,m}^{qp}(r_q, z) e^{im\theta_q} \tag{10}$$

Since the solution of the diffraction problem around multiple body configurations has been calculated by Mavrakos and Koumoutsakos (1987) and Mavrakos (1996), and the function $\Psi_{P,m}^{qq}$, from Eq.9, by Mavrakos and Konispoliatis (2011), the principal unknown of the problem is the function $\Psi_{P,m}^{qp}$. In order to express the potential in the form of Equation (10), Twersky's (1952) multiple scattering approach is implemented in the present formulation.

In doing so, we first assume that the isolated OWC device p $(p = 1,2,...,N)$ of the arrangement has an inner air pressure head different from the atmospheric one (i.e. close duct in the oscillating chamber), with the remaining devices being considered open to the atmosphere. In response to this pressure, the body p radiates its "zero order of radiation", ${}^{0}\phi_{7P}^{pp}$, given by Equation (9). Thus,

$$\sum_{p=1}^{N} {}^{0}\phi_{7P}^{pp} \tag{11}$$

is a first approximation to the total velocity potential radiated by the entire multiple–device configuration in the absence of interaction phenomena. The radiation potential, ${}^{0}\phi_{7P}^{pp}$, represents a "first order of excitation", ${}^{1}\phi_{0P}^{qp}, (q = 1,2,...,N; q \neq p)$ for each of the remaining devices of the arrangement in response to which they radiate a "first order of scattering" ${}^{1}\phi_{7P}^{qp}$ $(q = 1,2,...,N; q \neq p)$. The total "first order potential" around the body q due to inner air pressure in the body p, ${}^{1}\phi_{P}^{qp} = {}^{1}\phi_{0P}^{qp} + {}^{1}\phi_{7P}^{qp}$, $q \neq p$, has to satisfy the boundary conditions on the open-duct device in the q–th co-ordinate system. Especially, for the body p, with inner air pressure different to the atmospheric one, the "first order" incident and scattered, wave potentials, denoted by ${}^{1}\varphi_{0P}^{pp}$ and ${}^{1}\phi_{7P}^{pp}$, respectively, due to the remaining bodies of the arrangement vanish in the context of the present formulation. Indeed, as the remaining bodies of the arrangement are considered open to the atmosphere, they do not radiate any potential of "zero order" at all that would contribute to the "first order of excitation" of the p–th body, i.e.

$${}^{0}\phi_{7P}^{qp} = 0, \text{ for } q = 1, 2, \ldots, N, q \neq p \tag{12}$$

All the waves of the "first order of scattering" from the remaining open–duct devices can be considered as a "second order of excitation", ${}^{2}\phi_{0P}^{qp}$, for the body q $(q = 1,2,...,N)$, i.e.

$${}^{2}\phi_{0P}^{qp} = \sum_{\ell=1}^{N} \left(1 - \delta_{\ell q}\right) {}^{1}\phi_{7P}^{\ell p} \tag{13}$$

The q–th body radiates a wave of "second order of scattering" denoted by ${}^{2}\phi_{7P}^{qp}$. The total "second order potential" around the body q due to inner air pressure in the body p, ${}^{2}\phi_{P}^{qp} = {}^{2}\phi_{0P}^{qp} + {}^{2}\phi_{7P}^{qp}$, $q \neq p$, satisfies the boundary conditions on the open–duct device in the q–th coordinate system. The same conditions remain valid even if the body q coincides with the body p with inner air pressure because the inner and outer free surface boundary condition for body p has already been fulfilled through the solution ${}^{0}\phi_{7P}^{pp}$. For all the potentials of higher order interaction, ${}^{s}\phi_{P}^{pp}$, $s \geq 1$, it holds:

$$\omega^2 \, {}^{s}\phi_{p}^{pp} - g \frac{\partial^s \phi_{p}^{pp}}{\partial z} = \begin{cases} 0 & \text{for} \quad r_p \geq a_p \\ 0 & \text{for} \quad 0 \leq r_p \leq b_p \end{cases} \tag{14}$$

In this fashion we proceed to the s–th order of interaction for each device q $(q = 1,2,...,N)$ of the arrangement. The corresponding incident and total wave potentials will be respectively:

$${}^{s}\phi_{0P}^{qp} = \sum_{\ell=1}^{N} \left(1 - \delta_{\ell q}\right) {}^{s-1}\phi_{7P}^{\ell p} \tag{15}$$

$${}^{s}\phi_{P}^{qp} = {}^{s}\phi_{0P}^{qp} + {}^{s}\phi_{7P}^{qp} \tag{16}$$

Where ${}^{s}\phi_{P}^{qp}$, $s \geq 1$, satisfies the boundary condition on the open–duct device. Now, letting s approach infinity and summing over the various interaction orders, the total wave potential induced around the body q of the multiple–component configuration due to the air pressure head inside of body p, can be expressed as:

$$\phi_P^{qp}(r_q, \theta_q, z) = \phi_{0P}^{qp}(r_q, \theta_q, z) + \phi_{7P}^{qp}(r_q, \theta_q, z) = \sum_{s=1}^{\infty} {}^{s}\phi_{0P}^{qp}(r_q, \theta_q, z) +$$

$$\delta_{qp} \, {}^{0}\phi_{7P}^{pp}(r_p, \theta_p, z) + \sum_{s=1}^{\infty} {}^{s}\phi_{7P}^{qp}(r_q, \theta_q, z) =$$

$$= \sum_{s=1}^{\infty}\sum_{\ell=1}^{N}\left(1-\delta_{\ell q}\right)^{s-1}\phi_{7P}^{\ell p}\left(r_\ell,\theta_\ell,z\right)+\delta_{qp}\,{}^{0}\phi_{7P}^{pp}\left(r_p,\theta_p,z\right)+$$
$$+\sum_{s=1}^{\infty}{}^{s}\phi_{7P}^{qp}\left(r_q,\theta_q,z\right)=\delta_{qp}\,{}^{0}\phi_{7P}^{pp}\left(r_p,\theta_p,z\right)+\sum_{s=1}^{\infty}{}^{s}\phi_{P}^{qp}\left(r_q,\theta_q,z\right) \quad (17)$$

It is obvious from the above formulation that the total velocity potential, ϕ_P^{qp}, it is constructed in such a way that the imposed boundary conditions will be satisfied in the co–ordinate frame of the q–th, $\left(q=1,2,...,N\right)$, device. Indeed in cases where the examined device q coincides with the close–duct device p, the appropriate boundary condition on body p (Eq. 3) is satisfied by the term ${}^{0}\phi_{7P}^{pp}$ of the Equation (17). All the remaining terms of the above equation do not affect the validity of the imposed boundary condition since they are selected as solutions of the open - duct problem around any device of the arrangement.

Oppositely, if q is an open–duct device, the boundary condition (Eq. 3) in its coordinate system will also be satisfied from all the terms, ${}^{s}\phi_P^{qp}$, of the Equation (17) while the term, ${}^{0}\phi_{7P}^{pp}$, will be absent.

In this way the problem is reduced to the determination of the unknown potentials, ${}^{s}\phi_P^{qp}$, which by means of Equation (16) are expressed as a superposition of the s–th order wave scattered by the body q, ${}^{s}\phi_{7P}^{qp}$, and the $\left(s-1\right)$–th order waves scattered by the remaining bodies, ${}^{s-1}\phi_{7P}^{\ell p}$, $\left(\ell=1,2,...,N;\ell\neq q\right)$.

Each order of scattered wave potential ${}^{s}\phi_{7P}^{qp}$, $s\geq 1$, can be described in terms of cylindrical wave functions as [Mei, 1983; Mavrakos and Koumoutsakos, 1987]:

$${}^{s}\phi_{7P}^{qp}(r_q,\theta_q,z)=\frac{p_{in0}^{p}}{i\omega\rho}\sum_{m=-\infty}^{\infty}{}^{s}\Psi_{7P,m}^{qp}(r_q,z)e^{im\theta_q} \quad (18)$$

For the outer fluid domain of body q, i.e. $r_{q\geq}a_q$, $0\leq z\leq d$:

$${}^{s}\Psi_{7P,m}^{qp}(r_q,z)=\sum_{j=0}^{\infty}{}^{s}F_{Pm,j}^{qp}\frac{K_m(a_j r_q)}{K_m(a_j a_q)}Z_j(z) \quad (19)$$

Where K_m is the m–th order modified Bessel function of second kind, and $Z_j(z)$ are orthonormal functions in $\left[0,d\right]$ defined as follows:

$$Z_0(z)=N_0^{-1/2}\cosh\left(kz\right),\quad j=0 \quad (20)$$
$$Z_j(z)=N_j^{-1/2}\cos\left(a_j z\right),\quad j\geq 1 \quad (21)$$

Where

$$N_0=\frac{1}{2}\left[1+\frac{\sinh(2kd)}{2kd}\right] \quad (22)$$
$$N_j=\frac{1}{2}\left[1+\frac{\sin(2a_j d)}{2a_j d}\right] \quad (23)$$

Here k is the wave number related to ω through the dispersion equation:

$$\omega^2=kg\tanh(kd) \quad (24)$$

Whereas a_j are the real solutions of the equation:

$$\frac{\omega^2}{g}+a_j\tanh(a_j d)=0 \quad (25)$$

The s–th order scattering coefficients ${}^{s}F_{Pm,j}^{qp}$ (Eq.19) will be obtained through the solution of the respective order of diffraction problem –described by the velocity potential ${}^{s}\phi_P^{qp}$ – in the co–ordinate frame of body q. Once these coefficients, ${}^{s}F_{Pm,j}^{qp}$, have been determined, the potential, ${}^{s}\phi_P^{qp}$, around the body q, (q = 1,2, . . .,N), of the arrangement can be found by proper substitution of Equation (18) in (16).

However, since each of the scattered waves ${}^{s-1}\phi_{7P}^{\ell p}\left(r_\ell,\theta_\ell,z\right)$, $\left(\ell=1,2,...,N;\ell\neq p\right)$, contributing to ${}^{s}\phi_P^{qp}$, are expressed in terms of different co–ordinates, using a Bessel functions addition theorem, the velocity potentials expressed in the $\left(r_\ell,\theta_\ell,z\right)$ co–ordinates may be transformed to expressions in the reference co-ordinates $\left(r_q,\theta_q,z\right)$. From Watson (1966) it is confirmed that:

$$K_\nu(a_j r_\ell)e^{i\nu\theta_\ell}=\sum_{m=-\infty}^{\infty}(-1)^m K_{\nu-m}(a_j\ell_{\ell q})I_m(a_j r_q)e^{i(\nu-m)\theta_{\ell q}}e^{im\theta_q}$$

for $r_\ell<\ell_{\ell q}$ (26)

Where I_m denotes the m–th order modified Bessel function of first kind and $\ell_{\ell q}$ is defined in Fig.1.

From Abramowitz and Stegun (1970), it holds:

$$K_m(-ikr_q)=\frac{\pi}{2}i^{m+1}H_m(kr_q) \quad (27)$$

and

$$J_m(kr_q)=i^m I_m(-ikr_q)=i^m I_m(a_0 r_q) \quad (28)$$

for the imaginary root $a_0=-ik$. Thus, it can be obtained:

$$H_\nu(kr_\ell)e^{i\nu\theta_\ell}=\sum_{m=-\infty}^{\infty}H_{\nu-m}(k\ell_{\ell q})J_m(kr_q)e^{i(\nu-m)\theta_{\ell q}}e^{im\theta_q} \quad (29)$$

Using the above relations all terms of the s–th order velocity potential ${}^{s}\phi_P^{qp}$, described by Equation (16), are now expressed in the co–ordinate frame of the q–th body, i.e.

$${}^{s}\phi_P^{qp}(r_q,\theta_q,z)=\frac{p_{in0}^{p}}{i\omega\rho}\sum_{m=-\infty}^{\infty}{}^{s}\Psi_{P,m}^{qp}(r_q,z)e^{im\theta_q} \quad (30)$$

Where for the outer fluid domain, i.e. $r_{q\geq}a_q$, $0\leq z\leq d$ the function ${}^{s}\Psi_{P,m}^{qp}$ is given by:

$${}^{s}\Psi_{P,m}^{qp}(r_q,z)=\sum_{j=0}^{\infty}\left[{}^{s}G_{Pm,j}^{qp}\frac{I_m(a_j r_q)}{I_m(a_j a_q)}+{}^{s}F_{Pm,j}^{qp}\frac{K_m(a_j r_q)}{K_m(a_j a_q)}\right]Z_j(z) \quad (31)$$

where:

$$ {}^{s}G_{Pm,j}^{qp} = \sum_{i=1}^{N} (1-\delta_{\ell q}) \sum_{\nu=-\infty}^{\infty} i^{m+\nu} \frac{K_{\nu-m}(a_j \ell_{qp}) I_m(a_j a_q)}{K_\nu(a_j a_q)} {}^{s-1}F_{P\nu,j}^{(\ell)(p)} e^{i(\nu-m)\theta_{\ell q}} \qquad (32) $$

The first term in Equation (31) represents the contribution of the s–th order incident wave, ${}^{s}\phi_{0P}^{qp}$, to the potential ${}^{s}\phi_{P}^{qp}$, whereas the last term describes the scattered wave field of the corresponding order.

Using the expressions (30) and (31) the total wave field in the outer fluid domain, i.e. $r_q \geq a_q$, $0 \leq z \leq d$, given by Equation (17), can be written in the form of (10) with:

$$ \Psi_{P,m}^{qp}(r_q,z) = \delta_{qp}\Psi_{P,m}^{p}(r_p,z) + \sum_{j=0}^{\infty}\left[G_{Pm,j}^{qp}\frac{I_m(a_j r_q)}{I_m(a_j a_q)} + F_{Pm,j}^{qp}\frac{K_m(a_j r_q)}{K_m(a_j a_q)}\right] Z_j(z) \qquad (33) $$

Where:

$$ \Psi_{P,m}^{p}(r_p,z) = \sum_{j=0}^{\infty} F_{Pm,j}^{p} \frac{K_m(a_j r_p)}{K_m(a_j a_p)} Z_j(z) \qquad (34) $$

and

$$ G_{Pm,j}^{qp} = \sum_{s=1}^{\infty} {}^{s}G_{Pm,j}^{qp} \qquad (35) $$

$$ F_{Pm,j}^{qp} = \sum_{s=1}^{\infty} {}^{s}F_{Pm,j}^{qp} \qquad (36) $$

The first term in (33) represents the isolated device wave field around the p body due to its own internal pressure variation, the second term denotes the incident wave fields on body q emanating from the scattered fields of the remaining devices considered open and the last term is the scattered wave filed around the q –th device.

The corresponding expressions for the total velocity potential in the *III* and *M* fluid regions are:

$$ \phi_P^{III,qp}(r_q,\theta_q,z) = -i\frac{p_{in0}^{p}}{\omega\rho}\sum_{m=-\infty}^{\infty}\left[\sum_{n=0}^{\infty}\varepsilon_n\left[\left(F_{Pm,n}^{III,qp} + \delta_{qp}F_{P0,n}^{III,p}\right)R_{mn} + \left(F_{Pm,n}^{*III,qp} + \delta_{qp}F_{P0,n}^{*III,p}\right)R_{mn}^{*}\right]\cos\left(\frac{n\pi z}{h_q}\right)\right]e^{im\theta_q} \qquad (37) $$

$$ \phi_P^{M,qp}(r_q,\theta_q,z) = -i\frac{p_{in0}^{p}}{\omega\rho}\sum_{m=-\infty}^{\infty}\left[\delta_{qp}\, g_{P,m}^{p}(r_p,z) + \sum_{n=0}^{\infty}\left(F_{Pm,n}^{M,qp} + \delta_{qp}F_{P0,n}^{M,p}\right)\frac{I_m(a_n r_q)}{I_m(a_n b_q)}Z_n(z)\right]e^{im\theta_q} \qquad (38) $$

Where

$$ F_{Pm,n}^{III,qp} = \sum_{s=1}^{\infty} {}^{s}F_{Pm,n}^{III,qp} \text{ and } F_{Pm,n}^{M,qp} = \sum_{s=1}^{\infty} {}^{s}F_{Pm,n}^{M,qp} \qquad (39) $$

$$ R_{mn}(r_q) = \frac{K_m(\frac{n\pi b_q}{h_q})I_m(\frac{n\pi r_q}{h_q}) - I_m(\frac{n\pi b_q}{h_q})K_m(\frac{n\pi r_q}{h_q})}{I_m(\frac{n\pi a_q}{h_q})K_m(\frac{n\pi b_q}{h_q}) - I_m(\frac{n\pi b_q}{h_q})K_m(\frac{n\pi a_q}{h_q})} \qquad (40) $$

$$ R_{mn}^{*}(r_q) = \frac{I_m(\frac{n\pi a_q}{h_q})K_m(\frac{n\pi r_q}{h_q}) - K_m(\frac{n\pi a_q}{h_q})I_m(\frac{n\pi r_q}{h_q})}{I_m(\frac{n\pi a_q}{h_q})K_m(\frac{n\pi b_q}{h_q}) - I_m(\frac{n\pi b_q}{h_q})K_m(\frac{n\pi a_q}{h_q})} \qquad (41) $$

$$ R_{m0}(r_q) = \frac{\left(\frac{r_q}{b_q}\right)^m - \left(\frac{b_q}{r_q}\right)^m}{\left(\frac{a_q}{b_q}\right)^m - \left(\frac{b_q}{a_q}\right)^m} \quad R_{m0}^{*}(r_q) = \frac{\left(\frac{a_q}{r_q}\right)^m - \left(\frac{r_q}{a_q}\right)^m}{\left(\frac{a_q}{b_q}\right)^m - \left(\frac{b_q}{a_q}\right)^m} \qquad (42) $$

$$ g_{P,m}^{p}(r_p,z) = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases} \qquad (43) $$

and ε_n is the Neumann's symbol: $\varepsilon_0 = 1$, $\varepsilon_n = 2$, $n \geq 1$.

At this point we note that the form of the selected solutions for the velocity potentials in each fluid domain $\phi_P^{i,qp}, i = I, III, M$, is such that the boundary conditions at the horizontal boundaries of each fluid region and, in addition, the radiation condition at infinity in the outer fluid domain are a priori satisfied. Moreover, the potential functions $\phi_P^{i,qp}, i = I, III, M$ have been constructed in such a way that their homogeneous parts can be reduced to simple Fourier series at the vertical boundaries of adjacent fluid regions. This feature of the velocity potential representations facilitates essentially the solution procedure. The kinematic conditions at the body's vertical walls, as well as the requirement for continuity of the potential and its radial derivative at the vertical boundaries of neighboring fluid domains remain to be fulfilled.

Expressing these conditions an infinite system of linear equations for the determination of the unknown s–th order scattering coefficients, ${}^{s}F_{Pm,j}^{qp}$, in the fluid domain I of each device is obtained, can be written in follow general matrix form:

$$ [E]\left\{{}^{s}F_P^{I,qp}\right\} = [C_0] + \left\{D_P^{I,qp}\right\} \qquad (44) $$

where $\left\{D_P^{I,qp}\right\}$ denotes a complex vector whose elements are related to ${}^{s}G_{Pm,j}^{qp}$, given by Equation (32), $[E]$ is a square matrix its elements depend exclusively on the particular geometry of the examined device and the characteristics of the incident wave, and $[C_0]$ is a column matrix due to inner pressure in each device of the configuration. Having the Fourier coefficients in the outer fluid domain determined, the corresponding ones ${}^{s}F_P^{III,qp}, {}^{s}F_P^{M,qp}$, needed for the series representations of the potential functions in the *III* and *M* fluid region, respectively, can be calculated as well.

3 VOLUME FLOW

The time dependent volume flow produced by the oscillating internal water surface in *q* OWC device, *q*=1,2,…,*N*, is denoted by $Q^q(r_q,\theta_q,z;t) = \mathrm{Re}\left[q^q(r_q,\theta_q,z)\cdot e^{-i\omega t}\right]$

where:

$$q^q = \iint_{S_i^q} u_z dS_i = \iint_{S_i^q} u(r_q,\theta_q,z=d) r_q dr_q d\theta_q = \iint_{S_i^q} \frac{\partial \phi^q}{\partial z} r_q dr_q d\theta_q \quad (45)$$

Here u_z denotes the vertical velocity of the water surface, and S_i^q the inner water surface of the *q* device. It proves convenient to decompose the total volume flow, q^q, of the *q* device, same as for an isolated device, into two terms associated with the diffraction, q_D^q, and the pressure–dependent radiation problem, q_P^q, as follows:

$$q^q = q_D^q + q_P^q = q_D^q - p_{in0}^q\left(B^p - iC^P\right) \quad (46)$$

where B^q and C^q are the corresponding radiation conductance and susceptance, respectively. Assuming uniform pressure distribution inside the chamber, it can be shown that, even though all *m*-modes terms affect the values of the diffraction and radiation potentials, by substituting those potentials in Equation (45) only the modes with $m=0$ contribute to q_P^q and q_D^q, as in an isolated device [Mavrakos & Konispoliatis, 2011].

The volume flow associated with the diffraction and pressure radiation problem, is obtained as:

$$q_D^q = -i\frac{\omega^3}{g}\frac{H}{2} d 2\pi b_q \left(F_{D0,0}^{M,q} \frac{J_1(kb_q)}{kJ_0(kb_q)} N_0^{-1/2}\cosh(kd) + \right.$$
$$\left. + \sum_{n=1}^{\infty} F_{D0,n}^{M,q} \frac{I_1(a_n b_q)}{a_n I_0(a_n b_q)} N_n^{-1/2}\cos(a_n d) \right) \quad (47)$$

$$q_P^q = (-i\omega)\frac{p_{in0}^q}{\rho g} 2\pi b_q \left(F_{P0,0}^{M,qp} \frac{J_1(kb_q)}{kJ_0(kb_q)} N_0^{-1/2}\cosh(kd) + \right.$$
$$\left. + \sum_{n=1}^{\infty} F_{P0,n}^{M,qp} \frac{I_1(a_n b_q)}{a_n I_0(a_n b_q)} N_n^{-1/2}\cos(a_n d) \right) - \delta_{qp} i\omega \frac{p_{in0}^p}{\rho g} 2\pi b_p$$
$$\left(F_{P0,0}^{M,p} \frac{J_1(kb_p)}{kJ_0(kb_p)} N_0^{-1/2}\cosh(kd) + \sum_{n=1}^{\infty} F_{P0,n}^{M,p} \frac{I_1(a_n b_p)}{a_n I_0(a_n b_p)} N_n^{-1/2} \right.$$
$$\left. \cos(a_n d) \right) \quad (48)$$

Assuming that the Wells turbine is placed in a duct between the chamber and the outer atmosphere, of the *q* device, and it is represented by a pneumatic admittance Λ^q, then the total volume flow is equal to [Evans & Porter; 1996, Falnes; 2002]:

$$Q^q(t) = \Lambda^q \cdot P_{in}^q(t) \quad (49)$$

According to [Sarmento & Falcao; 1985, Falcao 2002], for the Wells turbine, can be obtained:

$$q^q = \left[\frac{KD}{\rho_a N} + (-i\omega)\frac{V_0}{\gamma P_a}\right] p_{in0}^q \quad (50)$$

Where *K* is constant for a given turbine geometry (independent of turbine size or rotational speed), *D* is turbine rotor diameter, *N* is the rotational speed (radians per unit time) and ρ_a, P_a are the atmospheric density and pressure.

4 WAVE FORCES

The various forces on the *q* oscillating water column device can be calculated from the pressure distribution given by the linearised Bernoulli's equation:

$$P(r_q,\theta_q,z;t) = -\rho\frac{\partial \Phi^q}{\partial t} = i\omega\rho\phi^q \cdot e^{-i\omega t} \quad (51)$$

Where ϕ^q is the *q* devices' velocity potential in each fluid domain *I*, *III* and *M*.

The horizontal and vertical exciting forces on the device have been presented in Mavrakos (1996), as the diffraction problem for an array of moonpool bodies is the same as for a group of OWC devices and thus, they will be no further elaborated here.

The hydrodynamic reaction forces and moments F_i^{qp} acting on the device *q* in the *i*–th direction due to pressure in the *p* device with inner pressure p_{in0}^p and frequency *ω*, can be obtained:

$$F_i^{qp} = f_i^{qp}\cdot e^{-i\omega t} = -\int\int_{S_0^q} i\omega\rho\phi_p^{qp}\cdot e^{-i\omega t}\cdot n_i^q dS \quad (52)$$

Where n_i^q are the generalized normal components defined by:

$$n^q = \left(n_1^q, n_2^q, n_3^q\right),\ r^q \times n^q = \left(n_4^q, n_5^q, n_6^q\right) \quad (53)$$

and r^q is the position vector of a point on S_0^q.

Furthermore, the complex force f_i^{qp} may be written in the form:

$$f_i^{qp} = \omega^2\left(e_i^{qp} + \frac{i}{\omega} d_i^{qp}\right)\cdot p_{in0}^p \quad (54)$$

Where e_i^{qp}, d_i^{qp} are the added mass and dumping coefficients, respectively.

5 NUMERICAL RESULTS

The calculation of the Fourier coefficients $F^{I}_{jm,i}$, $F^{III}_{jm,q}, F^{*III}_{jm,q}, F^{M}_{jm,i}$, j=D,P is the most significant part of the numerical procedure, because they affect the accuracy of solution. For the present calculations, in the first and the *M-th* ring element $i = 20$ terms were used, while for the third ring element $q = 40$. In addition, the presented results were obtained for five order interactions.

In figure 3 an array of three identical oscillating water column devices placed in a row is examined , for $a_q = 2b_q$, $d = 7.5b_q$, $d - h_q = (5/2)b_q$, and $\ell_{12} = \ell_{23} = 100b_q$ (for definitions see Fig.2.), in order to compare the dimensionless total vertical force on each body of the array with the results of Mavrakos & Konispoliatis (2012) work for a unit pressure head p^q_{in0}, $q = 1,2,3$, (vertical force in the device q due to its own internal pressure head and hydrodynamic reaction forces in vertical direction due to inner pressure in the remaining bodies). The incident wave is propagating to the positive x–axis at zero angle of incidence, the origin of the Cartesian co–ordinate system is on the sea bed and its vertical axis directed positive upward coinciding with the vertical axis of the second device local coordinate system.

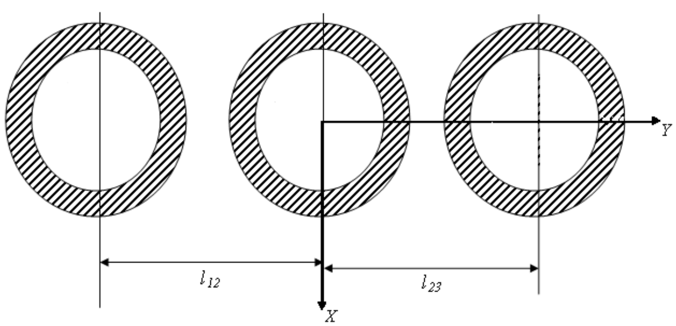

Figure 2. Array of three identical oscillating water column devices placed in a row

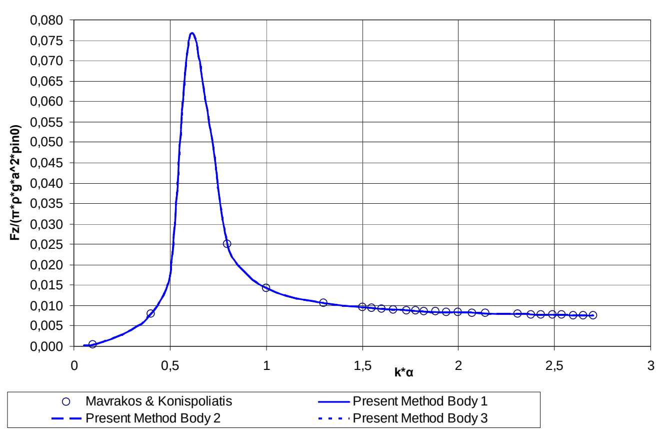

Figure 3. Total vertical force on each of three OWC devices due to a unit pressure head in each device

From Fig. 3 it is obvious that due to the large inter – body spacing, the hydrodynamic interaction phenomena do not affect the values of the vertical forces which in the particular case coincide with their isolated bodies counterparts. Figures 4, 5 present the hydrodynamic interaction coefficients in sway and heave, respectively, on the second (middle) body of a three OWC's array configuration (see Fig.2) with $\ell_{12} = \ell_{23} = 16b_q$, plotted against ka_q. The single body geometric characteristics are kept the same as previously, i.e. $a_q = 2b_q$, $d = 7.5b_q$, $d - h_q = (5/2)b_q$.

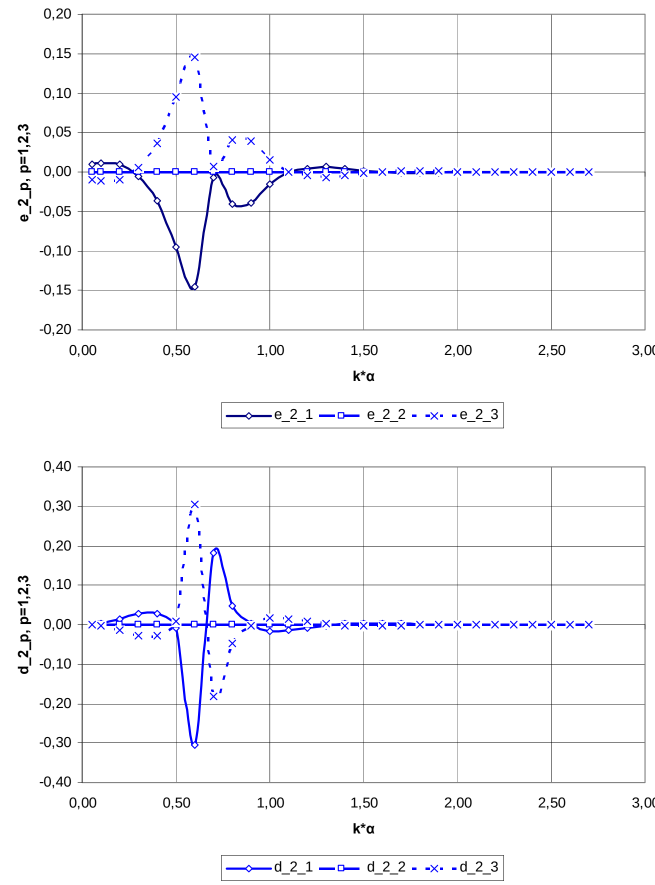

Figure 4. Hydrodynamic interaction coefficients in sway, e^{2p}_2, d^{2p}_2, p=1, 2, 3, versus ka_q

In figure 6 the modulus of the inner pressure for various turbine parameters $g^q_T = \dfrac{KD}{\rho_a N} = 1,3,6\left[m^5/(Ns)\right]$ versus ka_q, is plotted for an array of OWC devices placed in a row (see Fig. 2), for $d = (8/3)b_q$, $a_q = (4/3)b_q$, $h_q = 2b_q$, $\ell_{12} = \ell_{23} = (100/3)b_q$. It has been assumed, that all devices are characterized by the same g^q_T, q=1, 2, 3, value and from Equation (50) that $V_0 / \gamma P_a = 4.3\, m^5 / N$. The results are tested with those from Mavrakos & Konispoliatis (2011) work.

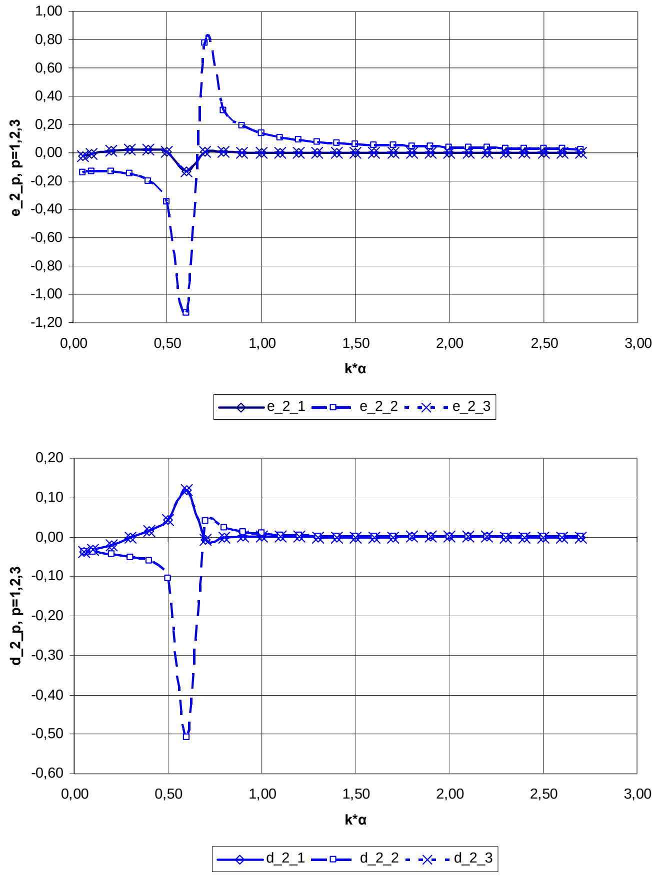

Figure 5. Hydrodynamic interaction coefficients in heave, e_3^{2p}, d_3^{2p}, p=1, 2, 3, plotted against ka_q

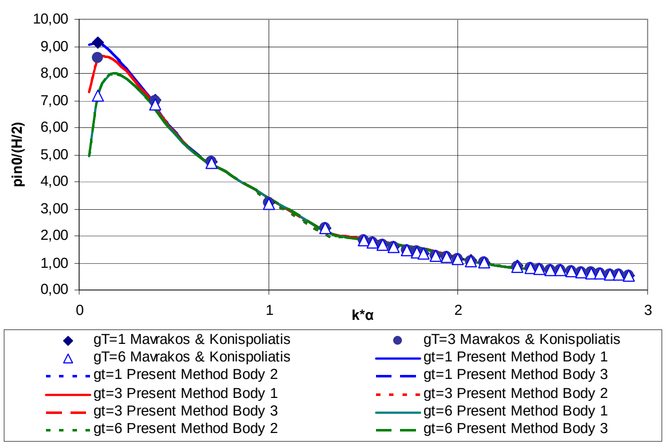

Figure 6. Modulus of inner pressure, [N/m^3], in each device of the configuration versus ka_q

Next the effect of the geometric arrangement and of the wave propagation on the internal device pressure is investigated, with the help of Fig. 7. The first two cases are concerned with the configuration shown in Fig. 2. Here, the three devices are placed in a row and the incident wave is propagating with angles of attack equal to 0^0 (first case) and 90^0 degrees (second case) with respect to the positive x – axis. The inter – body spacing is chosen equal to $\ell_{12} = \ell_{23} = 16 b_q$. In the third case, the geometric characteristics of the individual devices are kept the same as in figure 3, their geometrical arrangement though has been changed, by considering the three devices placed at the corners of a triangle, where $\ell_{12} = \ell_{23} = \ell_{13} = 16 b_q$, and the incident wave propagating along the positive x–axis at zero angle (Fig. 7). The dimensionless modules of the inner air pressures in each of the devices, for the three above mentioned cases are plotted in Figures 8, 9, 10 by assuming the same turbine parameter $g_T^q = \dfrac{KD}{\rho_a N} = 4\left[m^5/(Ns)\right]$, q=1, 2, 3.

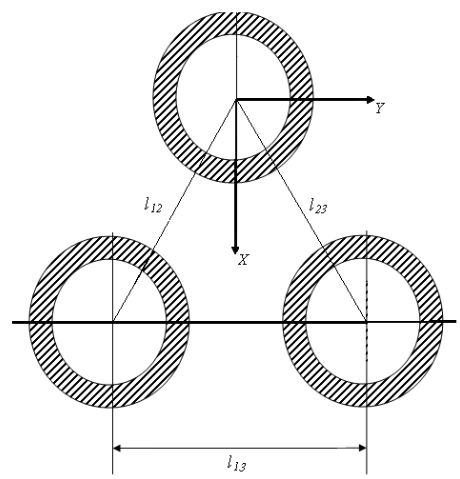

Figure 7. Array of three oscillating water column devices in triangular shape

6 CONCLUSIONS

An exact method has been presented for solving the diffraction and pressure radiation problems around an array of restrained OWC devices. The results of this analysis show the importance of the hydrodynamic interaction effects in evaluating the characteristics of such multiple – device arrangements. Both the hydrodynamic parameters and the characteristics of the turbine parameters have to be properly combined in order to improve the wave energy conversion.

The method is presently being extended to floating arrays of OWC devices in order to include the radiation problem due the motion of each body.

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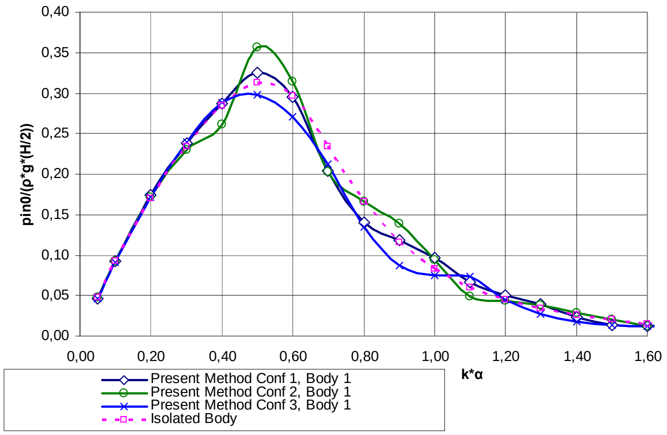

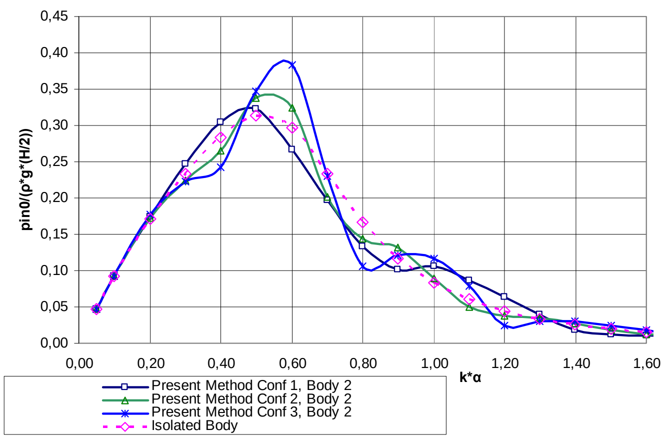

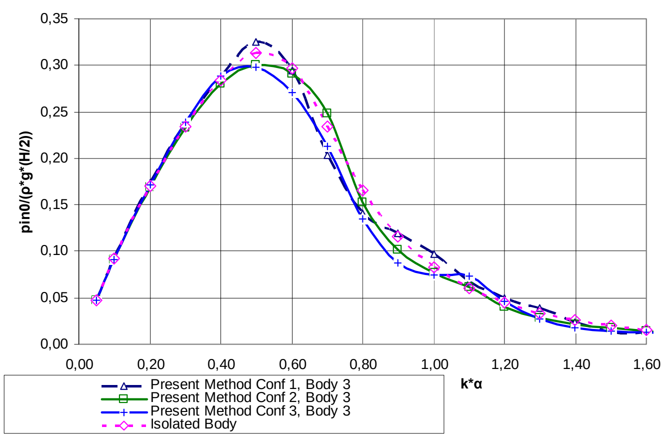

Figure 8, 9, 10. Modulus of inner pressure in each of the three devices for three different schematic configurations, versus ka_q

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